Advanced Computer Graphics

Intersection Acceleration

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Outline

- introduction
- bounding volume hierarchies
- uniform grids
- kd-trees
- octrees
- implementation

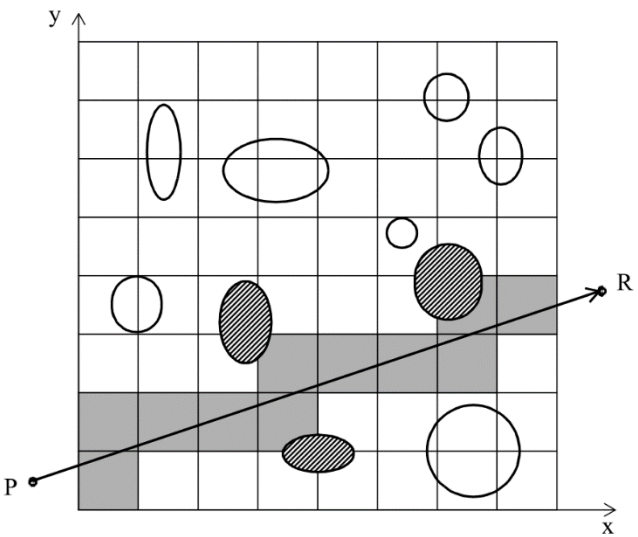
Motivation

- a large number of rays has to be checked for intersection with a large number of objects / shapes / primitives
- in 1968, Appel's approach spent up to 95% of the computation time for intersection computations
- use spatial data structures to accelerate the intersection computation
- minimize the number of potential intersection candidates by efficiently rejecting large parts of the scene geometry

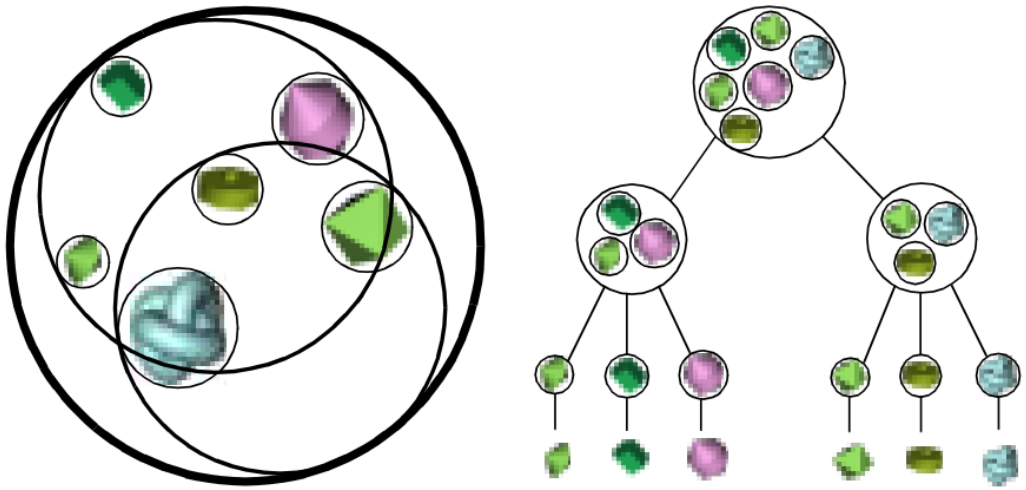
Spatial Data Structures

- bounding volumes
  - spheres
  - axis-aligned bounding boxes
  - object-oriented bounding boxes
  - k-dops
- bounding volume hierarchies
- space subdivision
  - uniform grids
  - octrees
  - k-d trees
  - BSP trees

Spatial Data Structures - Examples



space subdivision with a uniform grid
(space oriented)



bounding volume hierarchy with spheres
(object oriented)

[Allen Chang]

Spatial Data Structures - Efficiency

- determined by
  - generation
  - query
  - update
- generation is usually a pre-processing step
- query implements a traversal of the data structure to compute the first intersection of the scene with a ray
- update is only relevant for dynamic scenes
- static scenes: efficiency dominated by the query
- dynamic scenes: efficiency is determined by query and update

Spatial Data Structures - Applications

- e.g., ray tracing, collision detection in animation, neighborhood search in particle-based fluid animation
- generally, objects or primitives are represented in a spatial data structure to accelerate a certain query
- the generation of a particular spatial data structure is quite independent from the actual application
- the query of the data structure differs
  - first ray-primitive intersection in a raytracer
  - all primitive-primitive intersections for collision detection
  - all particles within a certain radius of a particle for fluid animation

Spatial Data Structures - Implementation

- depending on the query, different implementations can be appropriate
- e.g. for uniform grids
  - explicit representation
  - spatial hashing
  - compact hashing
  - index sort
  - z-index sort
- e.g. for bounding volume hierarchies
  - priority queues
  - skip lists

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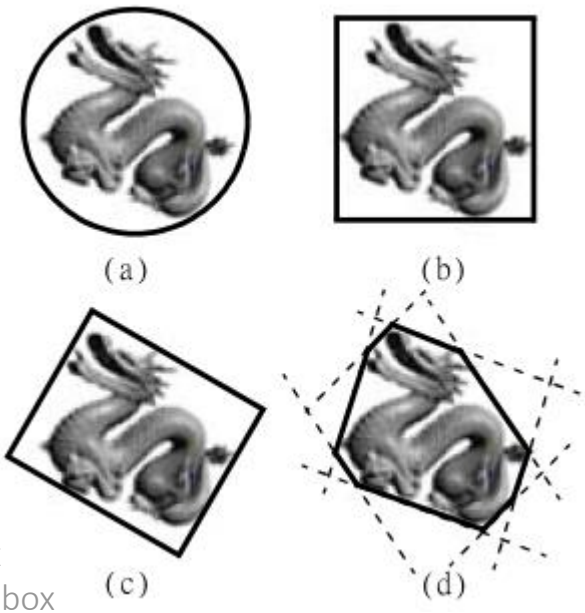
Bounding Volumes

- simple geometric shapes that enclose complex geometry
- if the simple geometry is not hit by the ray, then the complex geometry is also not hit by the ray
- for efficiency
  - intersection test should be fast
  - bounding volume should be tight-fitting
- a) $\Rightarrow$ d) improved fitting
- d) $\Rightarrow$ a) improved computation time

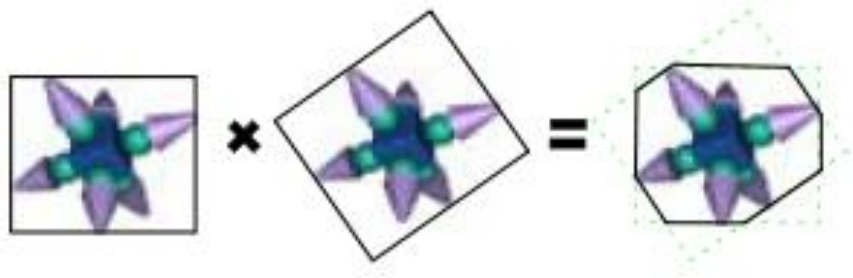


a) sphere
b) axis-aligned box
c) object-oriented box
d) slabs, k-DOP (discrete orientation polytope)

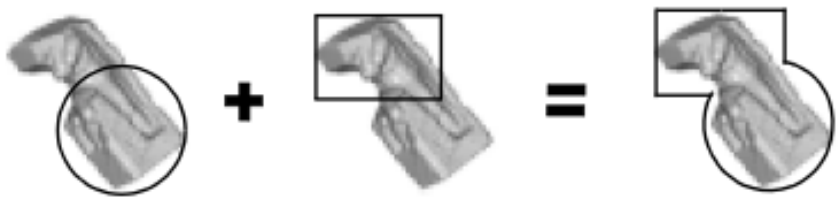
[Allen Chang]

Bounding Volumes

- intersection
  - if a ray hits all bounding volumes, the enclosed geometry is tested



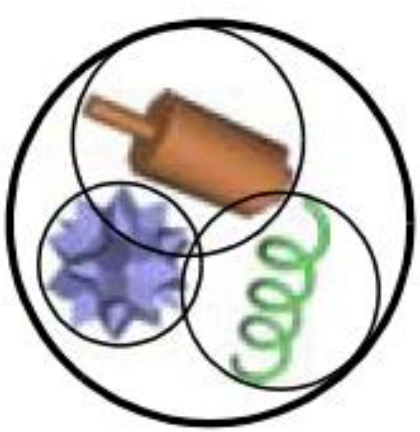
- union
  - if a ray hits one of the bounding volumes, the geometry is tested

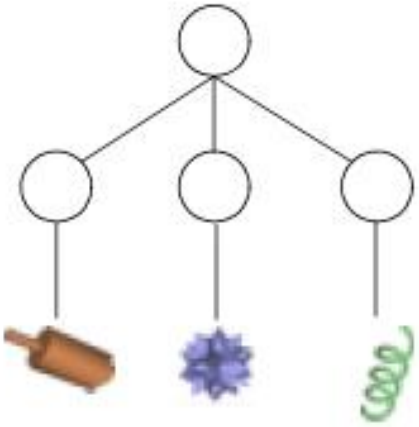


[Allen Chang]

Bounding Volume Hierarchies

- tree with a root node
- each node represents a bounding volume
- the union of all bounding volumes in a tree level encloses the entire geometry
- parts of the geometry (primitives) are stored in leaf nodes





[Allen Chang]

Bounding Volume Hierarchies - Construction

- degrees of freedom
  - top down / bottom up
  - number of children (branching factor)
- goals
  - balanced tree
  - minimal surface of bounding volumes per level
  - minimal overlap of bounding volumes per level
- strategies
  - group primitives close to each other (using sorted primitives)
  - split primitives into groups with similar numbers of primitives

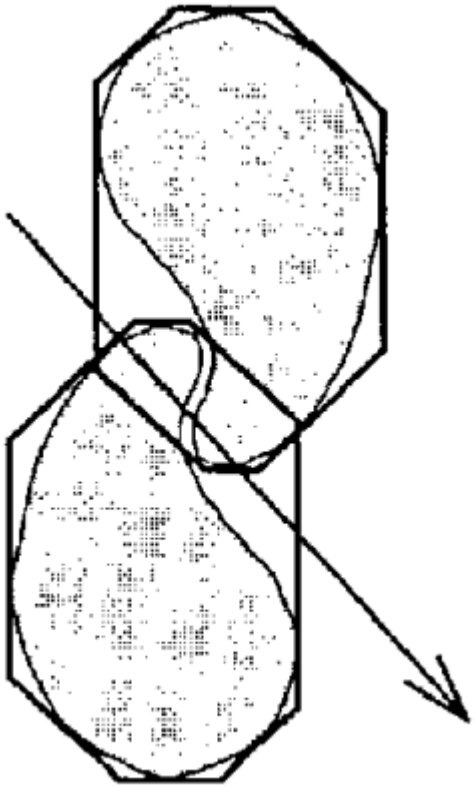
Bounding Volume Hierarchy
Ray Traversal - Priority Queue / Heap

- B - bounding volume of the root node
- heap - Min Heap / Priority List
-
```
heap = empty; intersection = inf;
if B.intersection(ray)<intersection then
heap.add(B);
while heap.notEmpty() and
heap.min.intersection(ray)<intersection do
   cand = heap.min; heap.remove (heap.min);
   if cand.leaf() then
      intersection = cand.minIntersection(ray);
   else
      foreach cand.child do
         if
cand.child.intersection(ray)<intersection then
            heap.add(cand.child);
return intersection;
```

depth-first, near-to-far

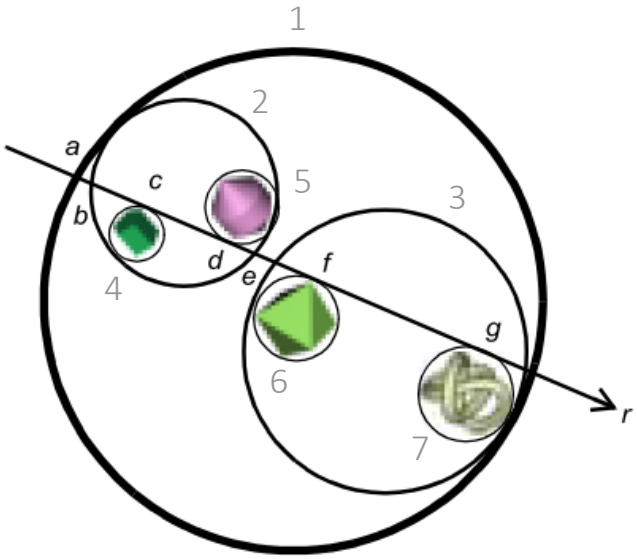
Bounding Volume Hierarchy - Ray Traversal

- if bounding volumes in one layer overlap, the closest intersection of a ray with a primitive is not necessarily in the bounding volume that is entered first

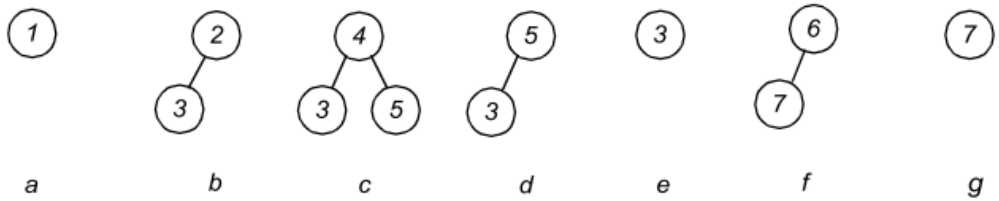


[Kay, Kajiya]

Bounding Volume Hierarchy
Ray Traversal - Priority Queue / Heap



(a)



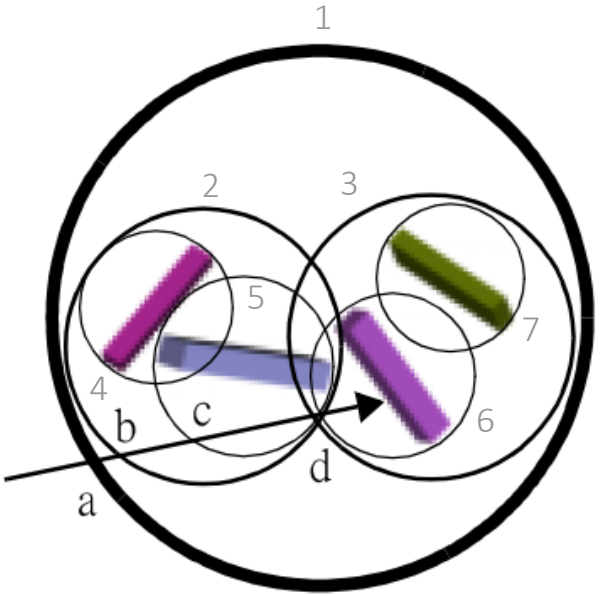
(c)

[Allen Chang]

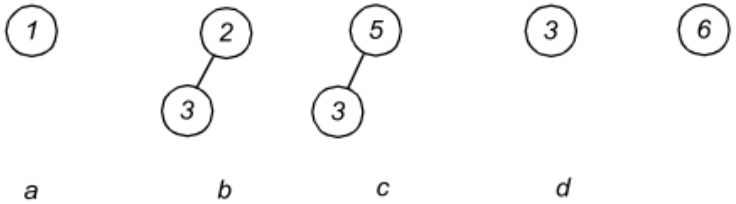
If an intersection would have been found within bounding volume 2, bounding volume 3 would not be processed, as the bounding volumes in this level of the hierarchy do not overlap.

Updating the heap is in O (log n). The tree-structure of the heap can be linearly represented with an array.

Bounding Volume Hierarchy
Ray Traversal - Priority Queue / Heap



(a)

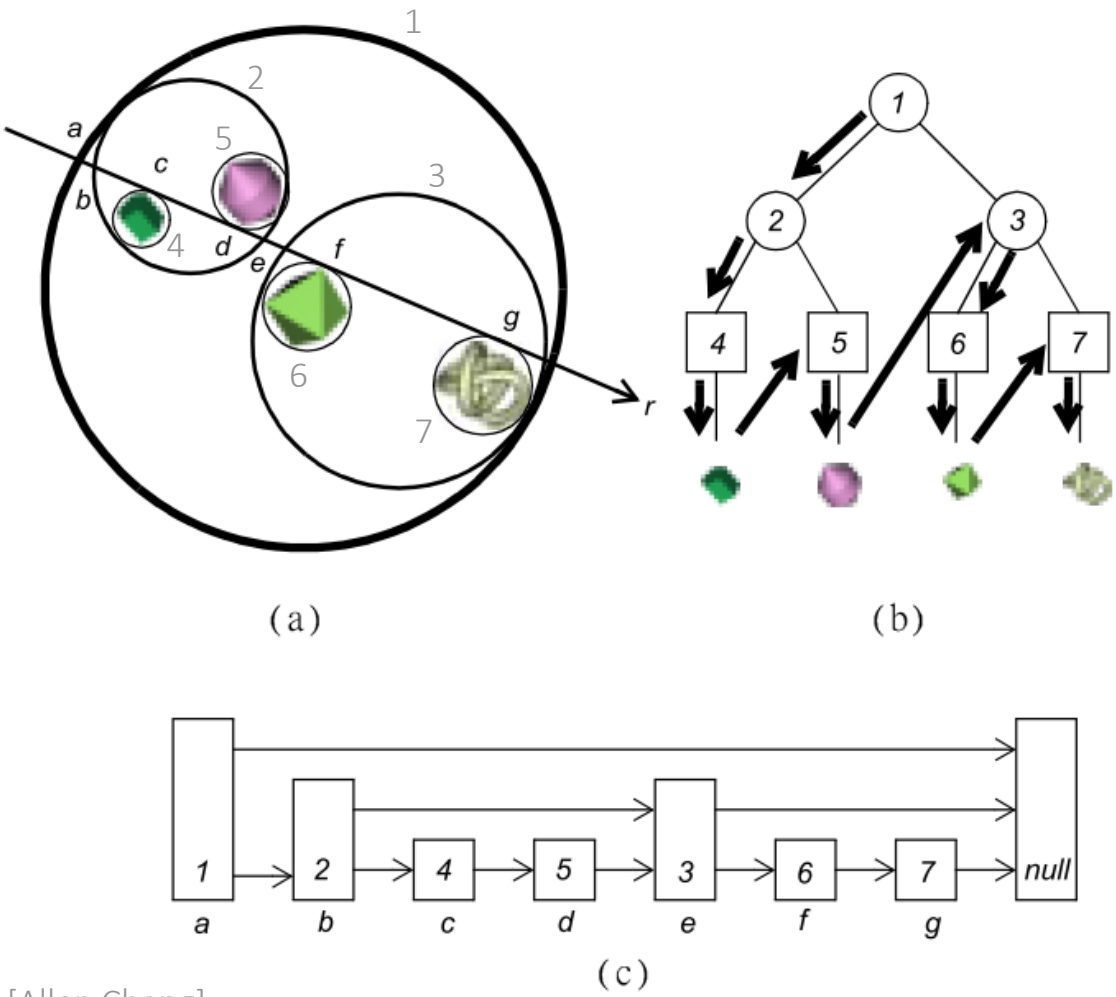


(c)

If an intersection would have been found within bounding volume 2, bounding volume 3 still has to be processed, as the bounding volumes in this level of the hierarchy overlap.

[Allen Chang]

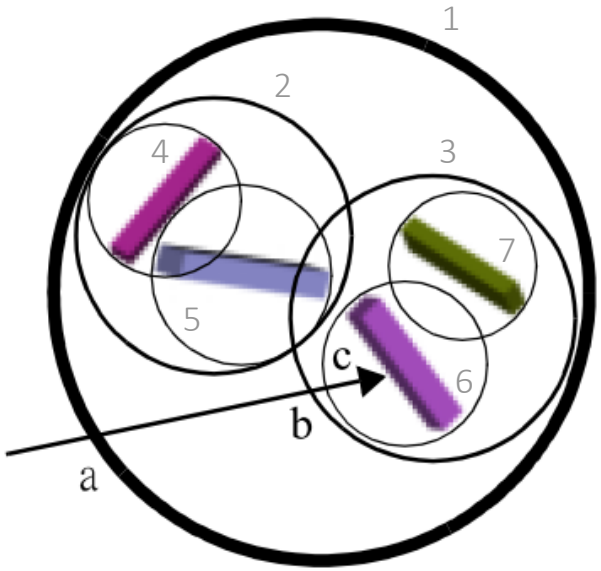
Bounding Volume Hierarchy
Ray Traversal - Skip List



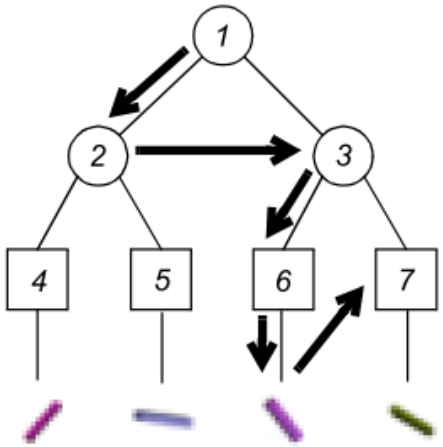
If (1) is hit, proceed with (2), otherwise proceed with (null). If (2) is hit, proceed with (4), otherwise proceed with (3)

[Allen Chang]

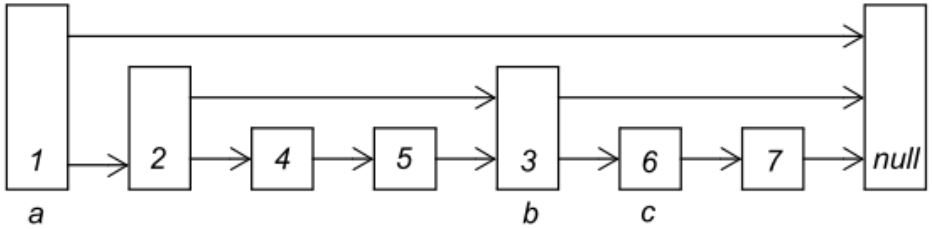
Bounding Volume Hierarchy
Ray Traversal - Skip List



(a)



(b)



(c)

[Allen Chang]

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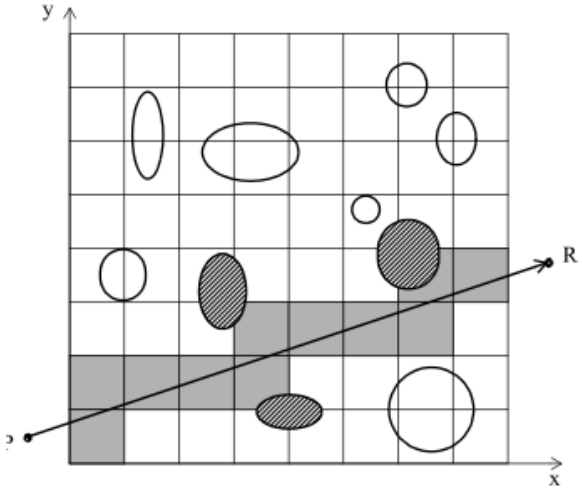
- uniform grids
- kd-trees
- octrees
- implementation

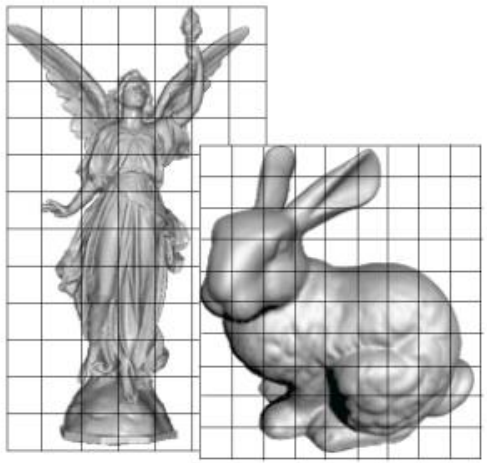
Uniform Grids - Construction

- subdivide 3D space into cubes / cuboids
- associate references to primitives that intersect a cell with this cell
- if a ray hits a cell, only the few primitives inside this cell are tested for intersection
- parameters
  - the cell size is essential for the performance
  - small cells: primitives are spread over many cells
  - large cells: too many primitives in one cell
  - number of cells often proportional to the number of primitives $M = \rho N$ with $2 \leq \rho \leq 10$

Uniform Grids - Variants



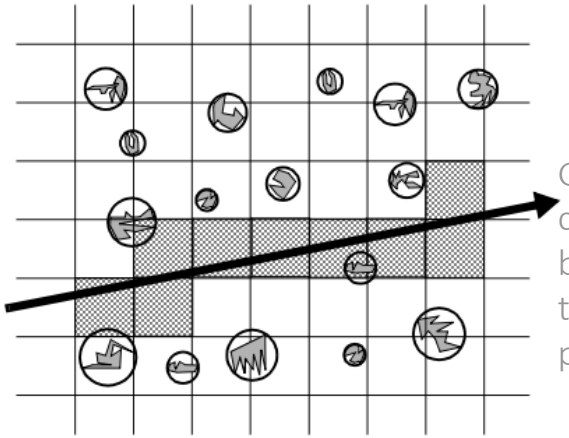
Basic concept: References to primitives are stored in cells



Several grids with varying resolution can be used in parallel

2	2	2	2	1	1	1	1
2	2	2	2	1	1	1	1
2	2	2	2	0		1	1
2	2	2	2	0	0	1	1
1	1			0	0		0
1	1	0	0	0			0
1	1	1	1				0
1	1	1	1	0	0	0	0

To accelerate the traversal, empty cells can be combined to larger areas (2^i x 2^i)

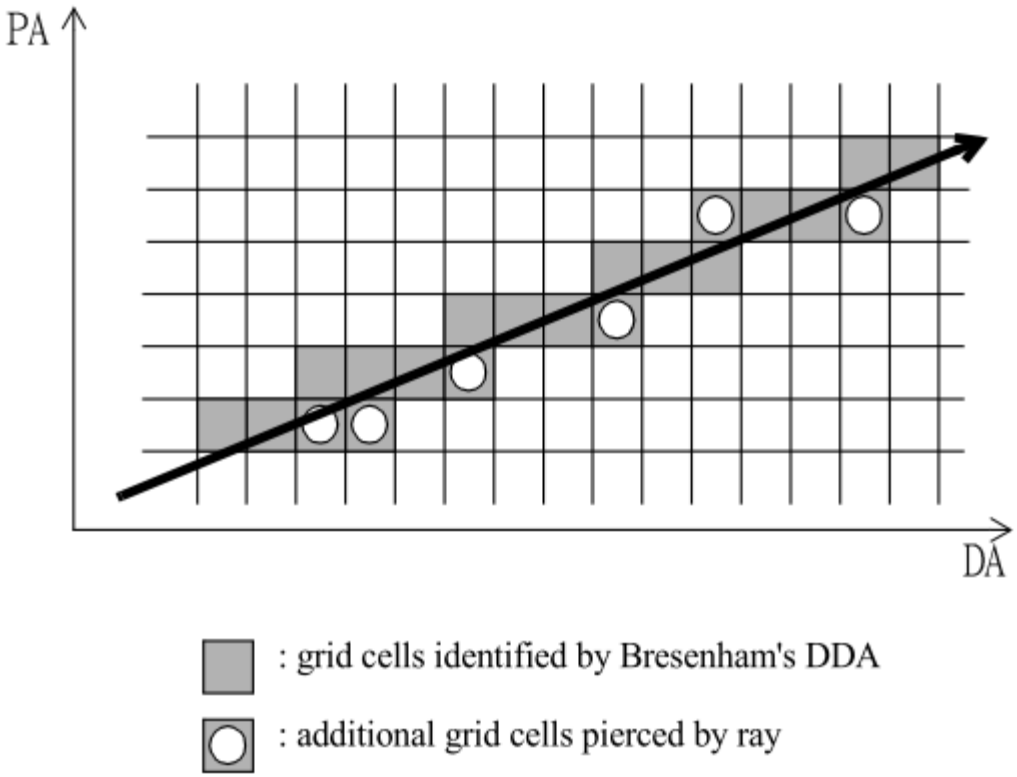


Grids are commonly combined with bounding volumes to simplify the primitive-cell mapping

[Allen Chang]

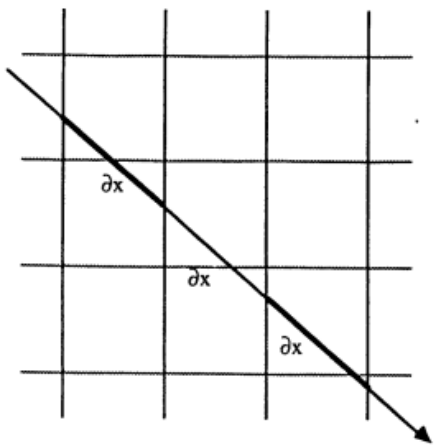
Uniform Grids - Traversal

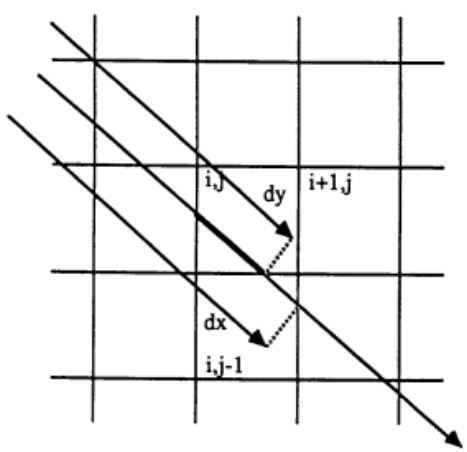
- unfortunately not as efficient as Bresenham's line algorithm



[Allen Chang]

Uniform Grids - Traversal





Simplified 2D algorithm

```
px=+1
py= -1
initialize ∂x,∂y,dx,dy,i,j
repeat
    if dx ≤ dy then
     begin
        i := i+px;
        dx:=dx+∂x;
    end;
    if dx ≥ dy then
    begin
        j:=j+py;
        dy:=dy+∂y;
    end;
until intersection in cell i,j;
```

$\partial x, \partial y, \partial z$

parametric distance along the ray between two grid planes perpendicular to x, y, z (infinite, if a ray is parallel to a principal axis of the grid)

dx, dy, dz

parametric value of the ray at the next intersection with a grid plane perpendicular to x, y, z (infinite, if a ray is parallel to a principal axis of the grid)

i, j, k

indices of the current grid cell

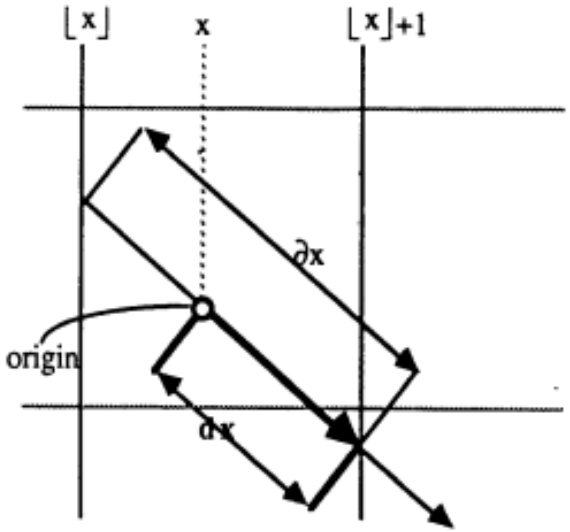
$p_x, p_y, p_z \in \{-1, 1\}$

increments of cell indices if a grid plane is intersected

[Cleary]

Uniform Grids
Traversal - Initialization

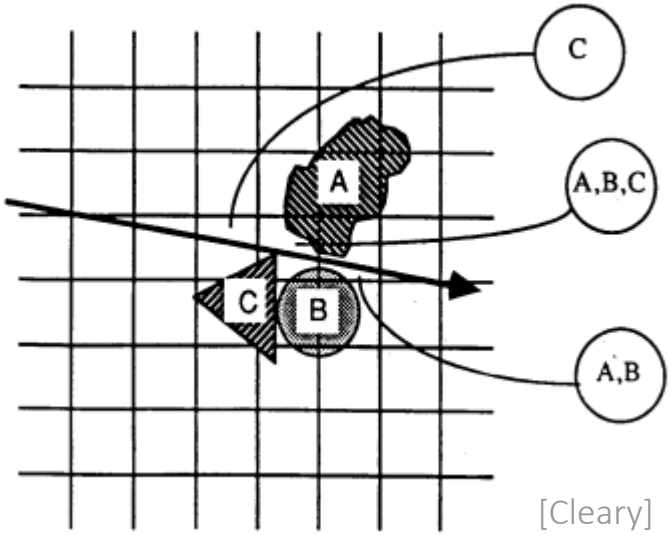
- dx, dy, dz have to be initialized with the first intersections of a ray with planes perpendicular to x, y, z inside the grid



[Cleary]

Uniform Grids - Mailboxing

- avoids multiple tests of one ray with the same object that is associated with different cells
  - each object stores a reference to the last ray that has been tested
- not necessarily useful for cheap intersection tests



Ray k is tested with C.
A reference to k is stored with C.

Ray k is tested with A, B. It is not tested with C.
A reference to k is stored with A.

Ray k is not tested with A, B,
as A and B store a reference to k.

[Cleary]

Uniform Grids - Robust Traversal

- standard traversal discards intersections outside the currently visited cell
  - the object is partially inside the cell, thus it is associated with the cell, but the actual intersection might be outside the cell
- standard traversal can miss intersections close at the cell border due to numerical issues
- ε-band might be used
- robust grid traversal
  - keep the closest intersection for a cell,
    even if the intersection is outside this cell
  - terminate ray traversal, if the maximum ray parameter of a cell is larger than the ray parameter of the closest intersection

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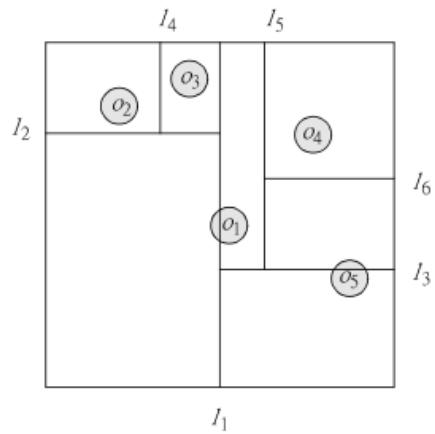
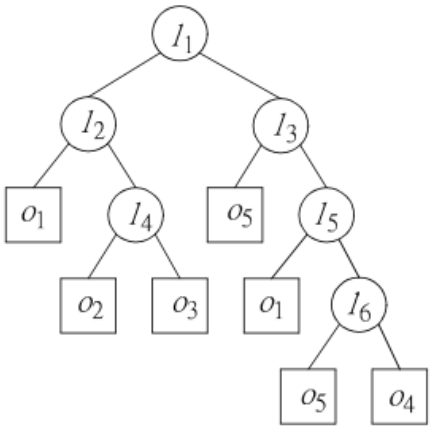
kd-Trees - Construction

- recursively subdivide 3D space in half spaces using planes
- planes are perpendicular to the x-, y-, z-axes
- steps
  - subdivide space with respect to the x-axis
  - subdivide the resulting subspaces with respect to y
  - continue with z, x, y, z, x, y, ... until there is only a small number of primitives in each subspace
- balanced tree ⇒ minimized depth

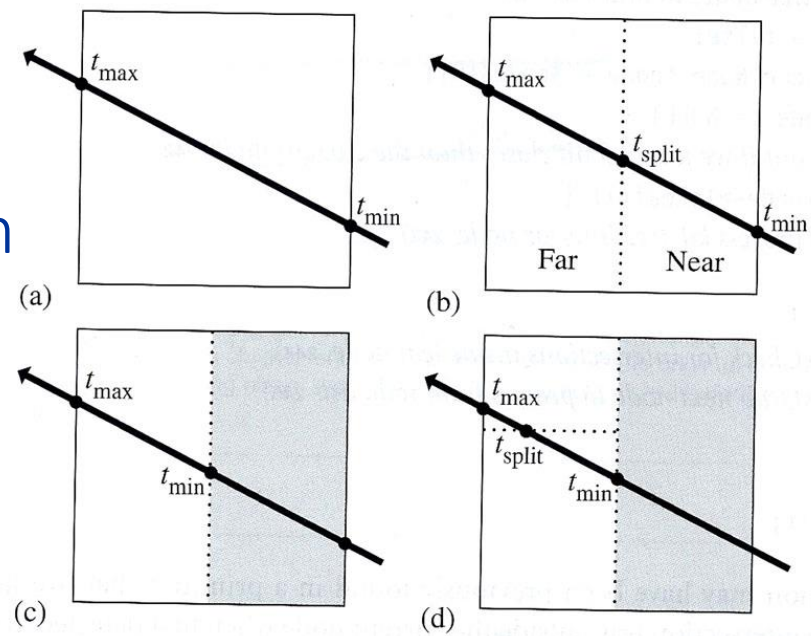




[Allen Chang]

kd-Trees - Traversal

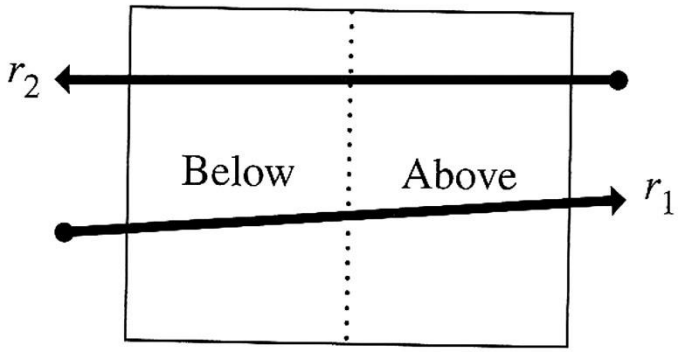
- check the global bounding box for intersection
- compute the intersection with the plane represented in the root node
- recursively traverse the two children in front-to-back order
- if a node is a leaf, check the primitives
- stop, if a ray-primitive intersection has been found or no further spaces have to be processed



[Pharr, Humphreys]

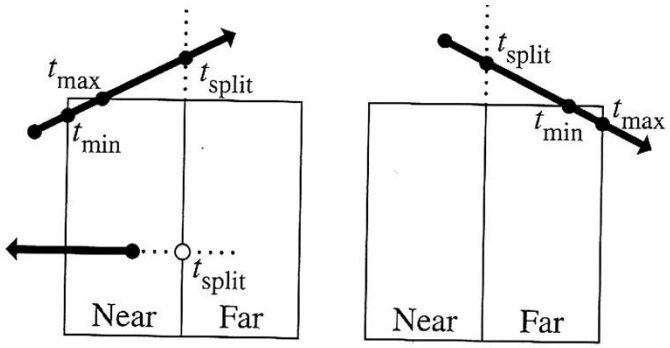
kd-Trees - Traversal

- given a position on the ray, the kd-tree can be traversed to decide the processing order of the subspaces



If a point on the ray is below with respect to a splitting plane, then the processing order is below-above, otherwise it's above-below.

- if the intersection with a splitting plane is outside the considered cuboid, the processing of half spaces can be accelerated



If t_{split} is outside $[t_{min}, t_{max}]$, the processing can be accelerated.

[Pharr, Humphreys]

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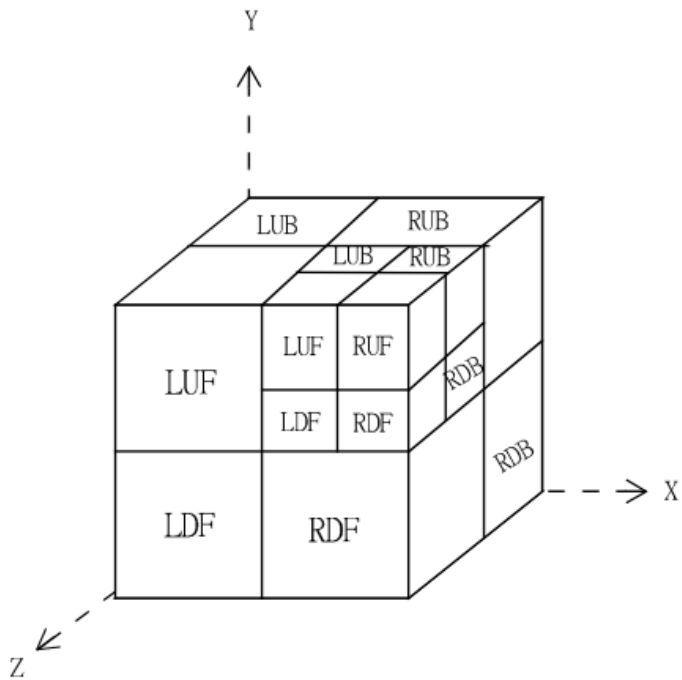
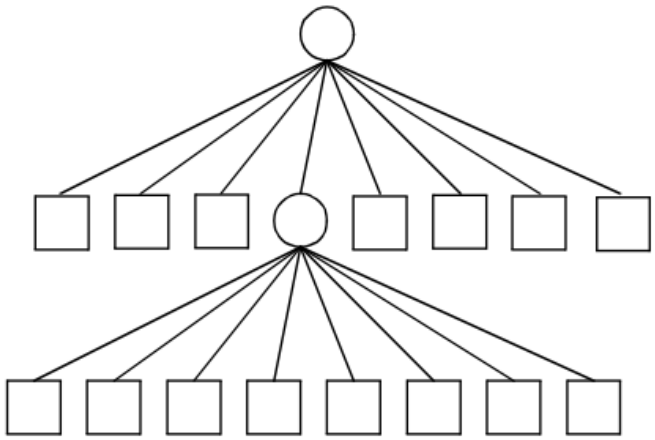
Octrees - Construction

- start with a global bounding box
- recursively subdivide the box using three planes into eight sub boxes until each sub box contains less than a certain number of primitives





[Allen Chang]

Octrees - Traversal

- e.g.
  - if the global bounding box is hit, process the octree in a BVH manner
  - as sub boxes in the same layer do not overlap, processing can be stopped if an intersection has been found and if the processing order of sub boxes is determined with a priority list

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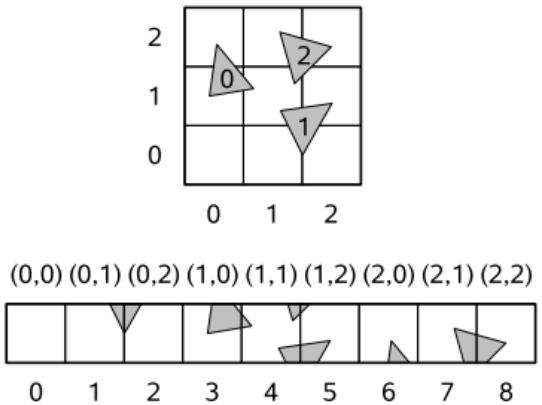
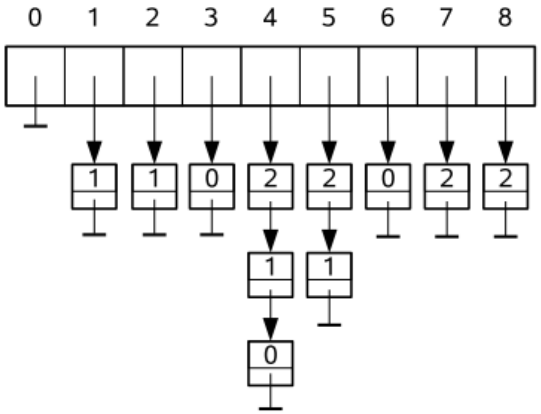
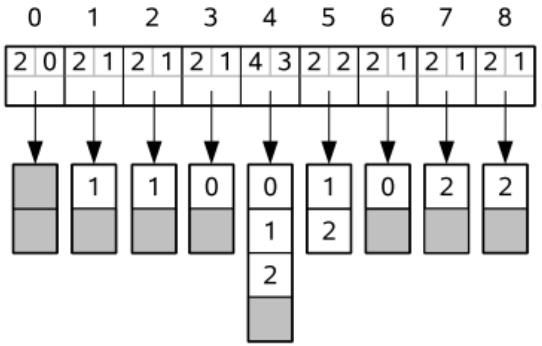
Uniform Grid
Linked List and Dynamic Array



2D grid and linearized representation



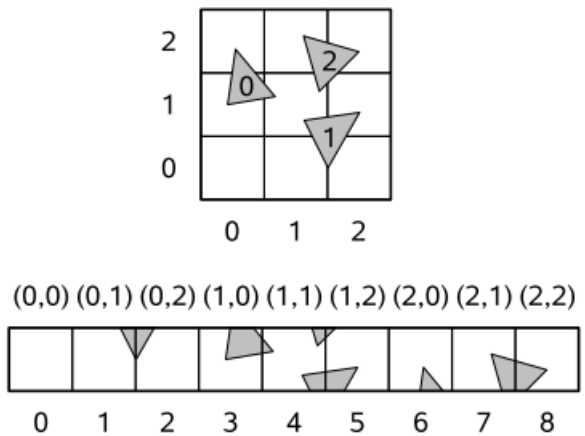
Linked list:
- minimal memory overhead
- bad locality
- frequent memory allocations
- single primitives can be inserted



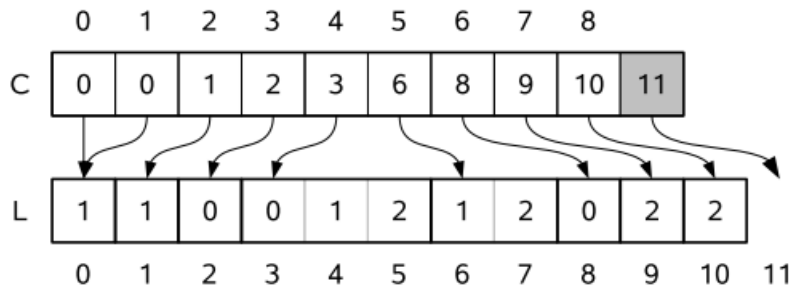
Dynamic array:
- keeps track of capacity and size
- less memory efficient
- improved data locality
- minimized memory allocations
- single primitives can be inserted

[Lagae]

Uniform Grid - Compact Grid



2D grid and linearized representation



offset of the corres-ponding primitive set

concatenation of all primitive sets

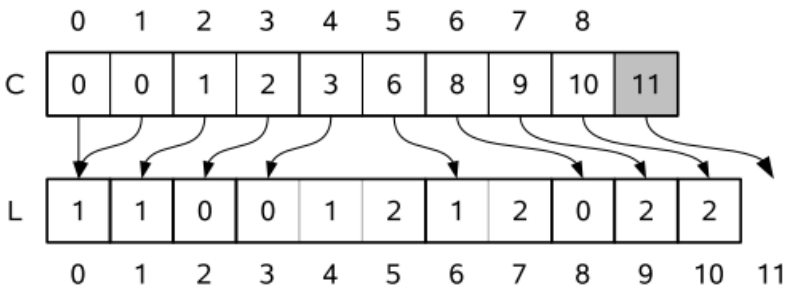
L[C[i]] - first primitive of cell i
C[i+1]-C[i] - number of primitives in cell i

Compact grid:
- two arrays
- useful for dynamic scenes
- generation rather expensive
- generation + traversal more efficient than linked lists or dynamic arrays
- single primitives cannot be inserted or deleted

[Lagae]

Uniform Grid - Compact Grid - Generation

- generate C
- store the number of primitives in each cell of C
  - loop over all primitives and increment the respective value in C
- accumulate the values in C
- generate L
- associate primitive i with cell j:
  L [--C [j]] = i
  - stores the primitives in reversed order into L
  - after insertion C contains the correct offsets



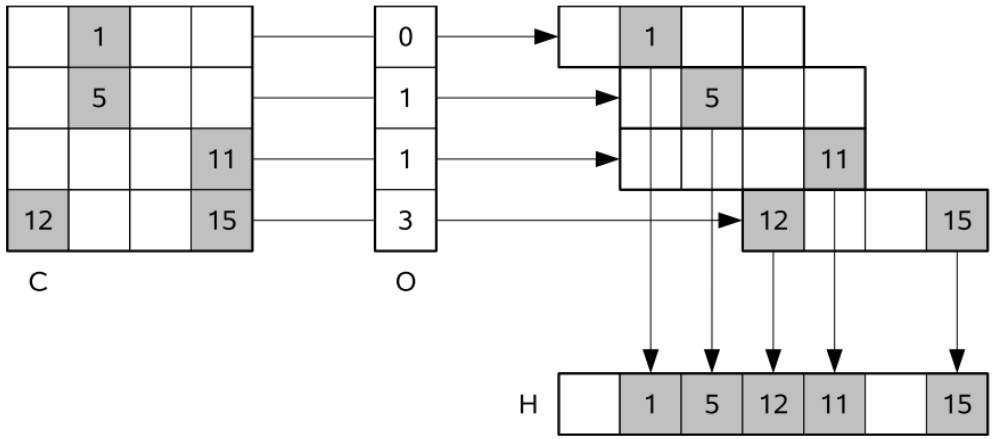
[Lagae]

Uniform Grid - Hashed Grid

- motivated by a large number of empty cells
- only stores filled grid cells in a hash table
- generation
  - process all primitives
  - compute the cell id
  - evaluate the hash function for the cell id
  - place the primitive into the hash table
    (use a dynamic array per hash table entry)
- traversal
  - compute the cell id that is intersected by a ray
  - evaluate the hash function for the cell id
  - look-up all relevant primitives from the hash table

Uniform Grid - Hashed Grid

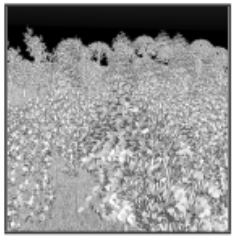
- to avoid hash collisions, perfect hashing can be used
- e.g., row displacement compression
  - identify occupied grid cells
  - compute an offset for each row (O) to store all rows with non-overlapping occupied cells in a hash table (H)



  - index into the hash table is O [row] + column

[Lagae]

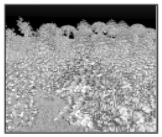
Uniform Grid - Performance

	Thai Statue 	Lucy 	Nature 
scene statistics			
#tri's	10.00 M	28.06 M	41.35 M
memory	343.32 MB	963.22 MB	1.39 GB
grid statistics			
grid res	302x508x261	485x278x832	906x202x902
# cells	40.04 M	112.18 M	165.08 M
% empty cells	98.44 %	99.00 %	92.04 %
avg # tri's / n-emp cell	29.25	41.50	28.52
avg # cells / tri	1.83	1.66	9.06
render statistics			
avg # n-emp cells / isect ray	2.38	2.06	12.12
avg # isect tests / isect ray	73.91	99.28	302.23

[Lagae]

Uniform Grid - Performance

- 1024 x 1024 x 1 rays
- diffuse shading

	Thai Statue 	Lucy 	Nature 
compact grid statistics			
mem cells	152.75 MB	427.93 MB	629.72 MB
mem tri lists	69.78 MB	178.06 MB	1.40 GB
build time	1.17 s	3.15 s	9.12 s
render time	1.55 s	1.90 s	10.75 s
time to image	2.72 s	5.05 s	19.87 s
memory	222.53 MB	605.98 MB	2.01 GB
hashed grid statistics			
data density	1.56 %	1.00 %	7.96 %
hash table size	967.47 K	1.76 M	28.82 M
hash table load factor	64.64 %	63.76 %	45.57 %
mem domain bits	4.77 MB	13.37 MB	19.68 MB
mem offset table	517.92 Kb	903.50 Kb	711.73 Kb
mem hash table	3.69 MB	6.73 MB	109.94 MB
mem cells	8.97 MB	20.98 MB	130.31 MB
compression ratio	1700 %	2040 %	483.24 %
mem tri lists	69.78 MB	178.06 MB	1.40 GB
build time	1.76 s	4.77 s	21.23 s
render time	1.43 s	1.53 s	10.07 s
time to image	3.18 s	6.30 s	31.30 s
memory	78.75 MB	199.04 MB	1.52 GB

[Lagae]

Summary

- spatial data structures minimize the number of potential intersection candidates by efficiently rejecting large parts of the scene geometry
  - bounding volumes
  - bounding-volume hierarchies
  - uniform grids
  - k-d trees
  - octress
- the actual implementation of the data structure significantly influences the performance