Advanced Computer Graphics
Radiosity

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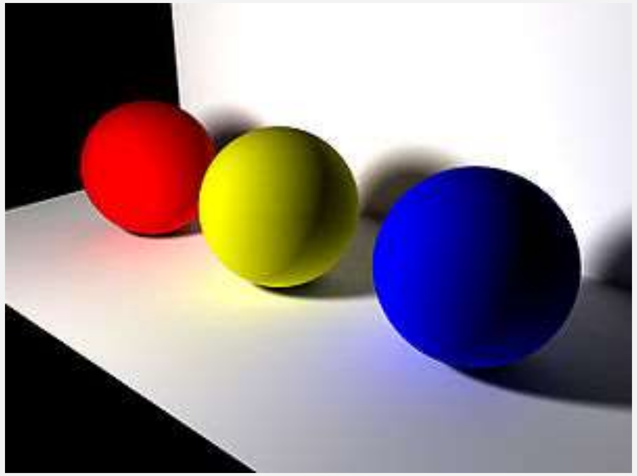
Motivation

- Rendering algorithm
- Global illumination approach
  - Global solution of a linear system
  - Considers global illumination (direct and indirect)
- View-independent solution
- Limited to Lambertian surfaces
- Conceptual basis for numerous rendering algorithms



Wikipedia: Radiosity (Computergraphik)

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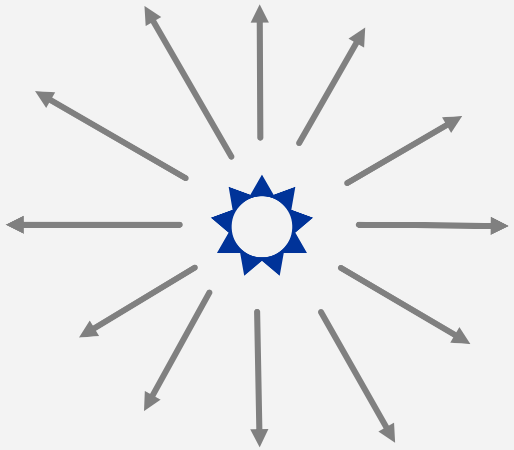
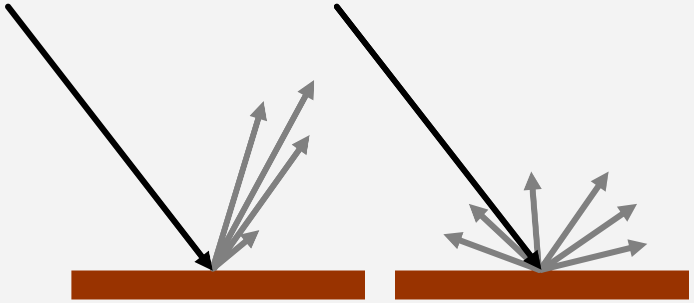
Outline

- Context
- Governing Equation
- System
- Solver
- Discussion

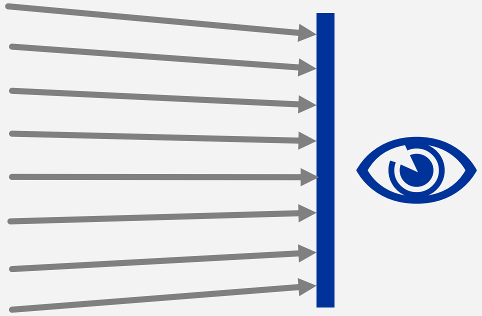
Towards Image Generation



Light is emitted
at light sources



Light is absorbed and
scattered at surfaces



Cameras
capture light

- Surfaces emit and / or reflect light
- Rendering algorithms compute light at sensors

Towards Image Generation

Light

↓

Material

↓

Rendering equation

↓

Solving the rendering equation

Radiosity – Rendering Algorithm

Light

↓

Material

↙

Rendering equation → Radiosity equation

↙

Solving the radiosity equation

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Notation

- Incident radiance onto a point
  $L_i(p, \omega_i), L(p \leftarrow \omega_i)$
- Exitant radiance from a point
  $L_o(p, \omega_o), L(p \rightarrow \omega_o)$
- BRDF at a point
  $f_r(p, \omega_i, \omega_o), f_r(p, \omega_i \leftrightarrow \omega_o)$

Reflectance Equation

- Relation between irradiance and exitant radiance

$$dL(p \rightarrow \omega_o) = f_r(p, \omega_i \leftrightarrow \omega_o)\ dE(p \leftarrow \omega_i)$$ BRDF definition

- Irradiance is induced by radiance

$$dL(p \rightarrow \omega_o) = f_r(p, \omega_i \leftrightarrow \omega_o)\ L(p \leftarrow \omega_i)\ \cos(\omega_i, n_p)\ d\omega_i$$

- Integration over the hemisphere ⇨ reflectance equation

$$L(p \rightarrow \omega_o) = \int_\Omega f_r(p, \omega_i \leftrightarrow \omega_o)\ L(p \leftarrow \omega_i)\ \cos(\omega_i, n_p)\ d\omega_i$$

- Reflectance equation establishes a relation between incident and exitant radiance

Reflectance vs. Rendering Equation

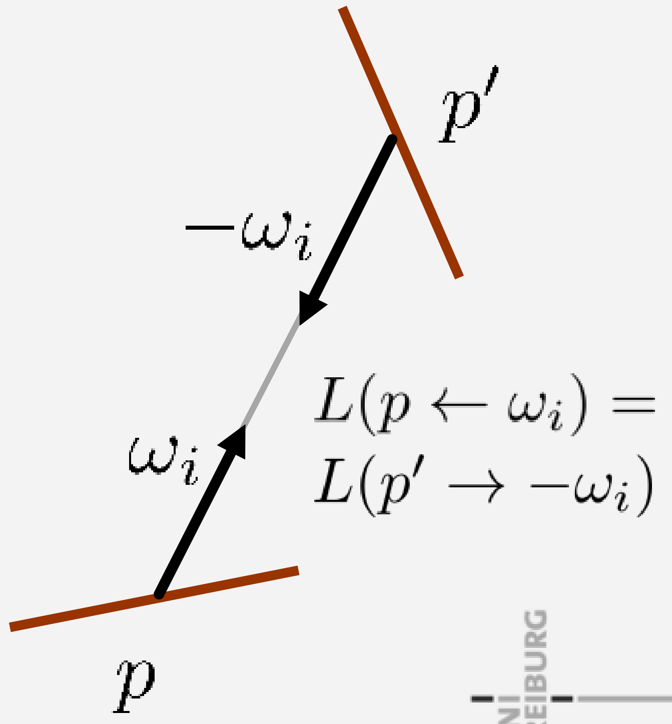
- Reflectance equation relates incident and exitant radiance at surfaces
- Rendering equation incorporates emissive surfaces, i.e. light sources

$$L(p \rightarrow \omega_o) = L_e(p \rightarrow \omega_o) + \int_\Omega \, f_r(p, \omega_i \leftrightarrow \omega_o) \, L(p \leftarrow \omega_i) \, \cos(\omega_i, n_p) \, d\omega_i$$

- Exitant radiance is the sum of reflected and emitted radiance

Ray-Casting Operator

- Incident radiance $L(p \leftarrow \omega_i)$ at a point p is equal to the exitant radiance $L(p' \rightarrow -\omega_i)$ from another point p'
- Ray-casting operator $p' = r_c(p, \omega_i)$
  - Nearest intersection from p into direction ω_i
- Radiance
  - $L(p \leftarrow \omega_i) = L(r_c(p, \omega_i) \rightarrow -\omega_i)$
  - $L(p \leftarrow \omega_i) = L(p' \rightarrow -\omega_i)$
  - If $r_c(p, \omega_i)$ does not exist, $L(p \leftarrow \omega_i)$ is user-defined, e.g. emission from sky



Rendering Equation

- $L(p \rightarrow \omega_o) = L_e(p \rightarrow \omega_o) + \int_\Omega f_r(p, \omega_i \leftrightarrow \omega_o) L(p' \rightarrow -\omega_i) \cos(\omega_i, n_p) d\omega_i$
- Establishes relations between exitant radiances
- Expresses the steady state of radiances in a scene
- Governs the computation of exitant radiances from all scene points into all directions

[Akenine-Möller et al.]

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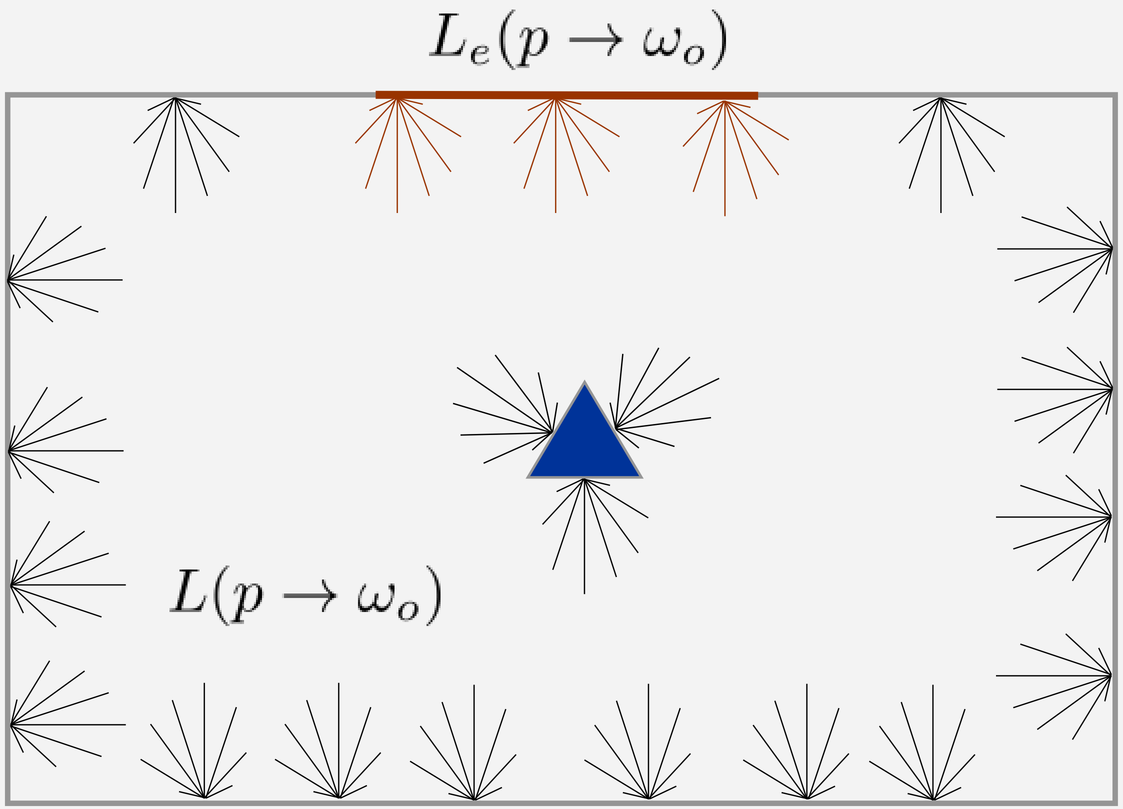
Solution of the Rendering Equation

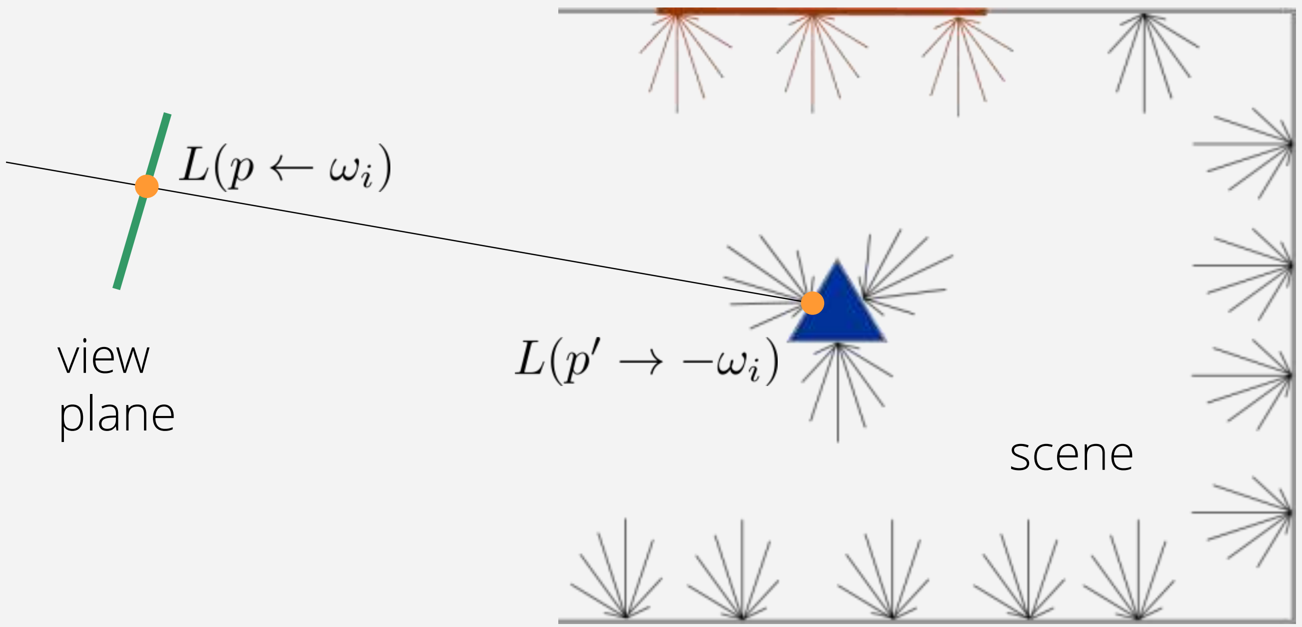
– Exitant radiances from all scene points into all directions



Cornell box

Rendering of the Solution

- At an arbitrarily placed and oriented sensor
  - Cast a ray through position p in an image plane into direction ω_i
  - Lookup $L(p \leftarrow \omega_i) = L(r_c(p, \omega_i) \rightarrow -\omega_i) = L(p' \rightarrow -\omega_i)$



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Simplified Setting

- Lambertian material
  - Exitant radiance independent from direction
  - Radiance into arbitrary direction can be computed from radiosity $L(p \rightarrow \omega_o) = \frac{B(p)}{\pi}$
- Discretized scene representation with faces, e.g., triangles
  - Assume constant radiosity per face

⇨ Problem is simplified to n radiosity values for n faces

⇨ n instances of the rendering equation govern the solution

Goal - System of Governing Equations

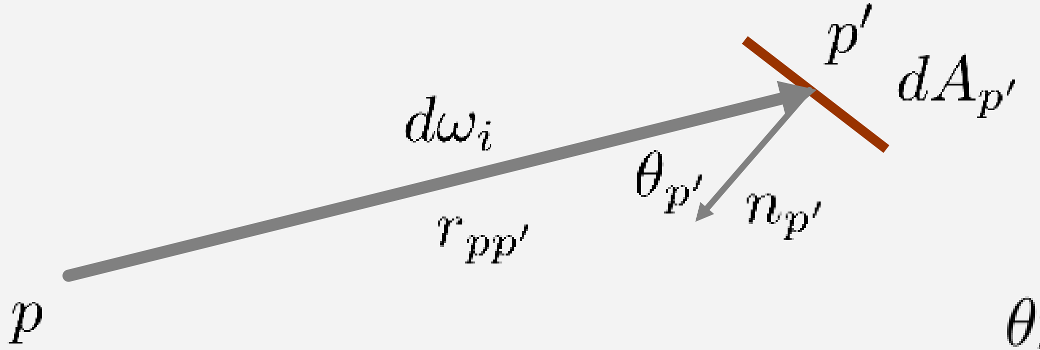
- Simplified setting results in a linear system with unknown radiosity values B_i at faces

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn-1} & a_{nn} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix}$$

- Radiance B_i at face i depends on radiances B_j at faces j which are visible from face i

Hemispherical and Area Form

- Differential solid angle corresponds to a differential surface area
- If an infinitesimally small area $dA_{p'}$ at position p' converges to zero, then the solid angle $d\omega_i$ also converges to zero and the relation $d\omega_i = \frac{\cos(-\omega_i, n_{p'})}{r^2_{pp'}} dA_{p'}$ is correct in the limit



$$\theta'_p = \cos(-\omega_i, n_{p'})$$

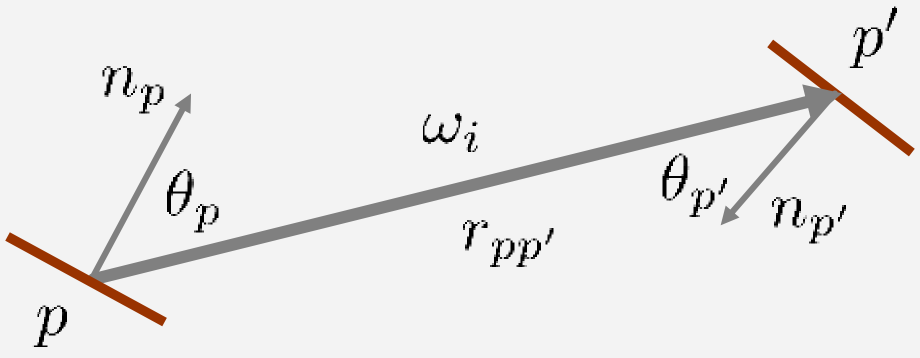
Hemispherical and Area Form

– Hemispherical form of the rendering equation

$$L(p \to \omega_o) = L_e(p \to \omega_o) + \int_\Omega f_r(p, \omega_i \leftrightarrow \omega_o) L(p' \to -\omega_i) \cos(\omega_i, n_p) d\omega_i$$

can be written in area form

$$L(p \to \omega_o) = L_e(p \to \omega_o) + \int_S f_r(p, \omega_i \leftrightarrow \omega_o) L(p' \to -\omega_i) \frac{\cos(\omega_i, n_p) \cos(-\omega_i, n_{p'})}{r_{pp'}^2} dA_{p'}$$



$$\theta_p = \cos(\omega_i, n_p)$$

$$\theta_p' = \cos(-\omega_i, n_{p'})$$

Area Form of the Rendering Equation

- Integral over all differential surface areas $p' = r_c(p, \omega_i)$ obtained from the ray-casting operator

$$L(p \to \omega_o) = L_e(p \to \omega_o) + \int_S f_r(p, \omega_i \leftrightarrow \omega_o) L(p' \to -\omega_i) \frac{\cos(\omega_i, n_p)\cos(-\omega_i, n_{p'})}{r_{pp'}^2} dA_{p'}$$

- Integral over all differential surface areas x of a scene

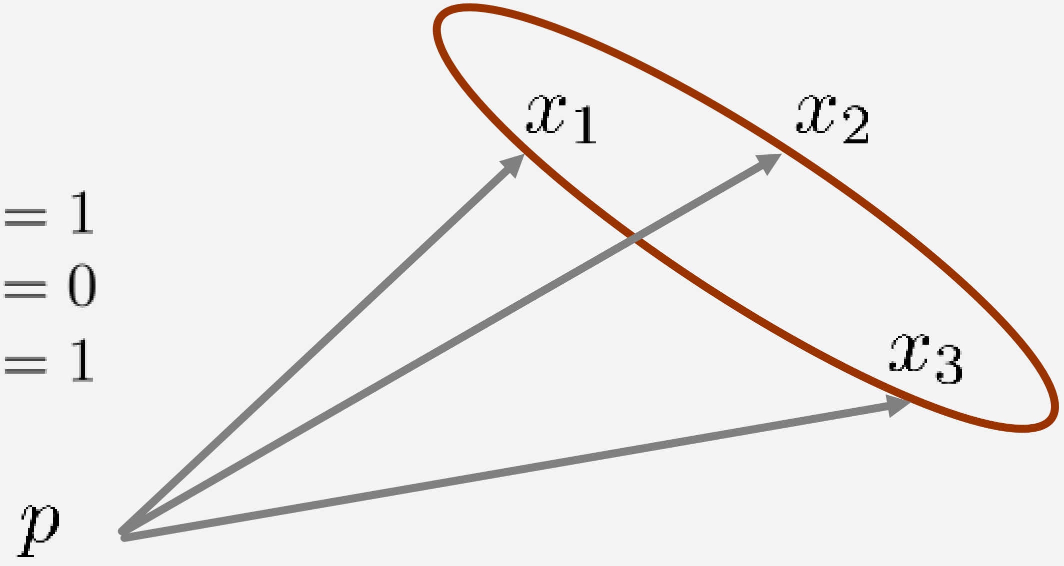
$$L(p \to \omega_o) = L_e(p \to \omega_o) + \int_S f_r(p, \omega_i \leftrightarrow \omega_o) L(x \to -\omega_i) V(p, x) \frac{\cos(\omega_i, n_p)\cos(-\omega_i, n_x)}{r_{px}^2} dA_x$$

visibility function

Visibility Function

- $\int_S f_r(p, \omega_i \leftrightarrow \omega_o) L(x \rightarrow -\omega_i) V(p, x) \frac{\cos(\omega_i, n_p)\cos(-\omega_i, n_x)}{r_{px}^2} dA_x$
- Position x contributes to the integral, if it is visible from p

$$V(p, x_1) = 1$$
$$V(p, x_2) = 0$$
$$V(p, x_3) = 1$$



Radiosity Integral Equation

– Rendering equation

$$L(p \to \omega_o) = L_e(p \to \omega_o) + \int_S f_r(p, \omega_i \leftrightarrow \omega_o) L(x \to -\omega_i) V(p, x) \frac{\cos(\omega_i, n_p)\cos(-\omega_i, n_x)}{r_{px}^2} dA_x$$

– Radiance can be computed from radiosity for Lambertian surfaces: $L(p \to \omega_o) = \frac{B(p)}{\pi}$

– Radiosity equation

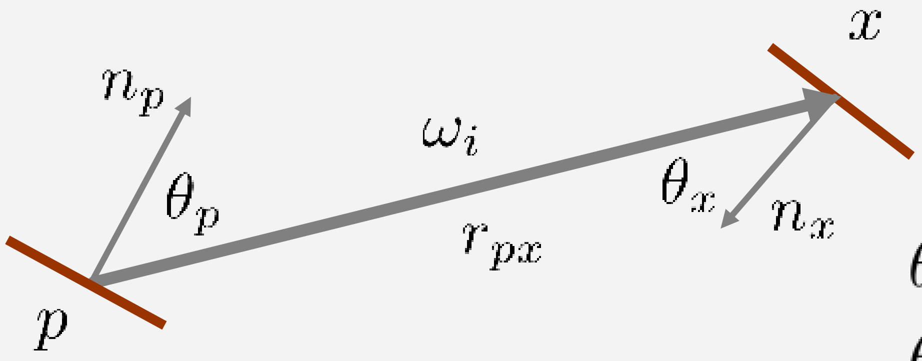
$$B(p) = B_e(p) + \int_S f_r(p, \omega_i \leftrightarrow \omega_o) B(x) V(p, x) \frac{\cos(\omega_i, n_p)\cos(-\omega_i, n_x)}{r_{px}^2} dA_x$$

$$B(p) = B_e(p) + \frac{\rho(p)}{\pi} \int_S B(x) V(p, x) \frac{\cos(\omega_i, n_p)\cos(-\omega_i, n_x)}{r_{px}^2} dA_x$$

Constant BRDF for Lambertian surfaces

Kernel

- $K(p,x) = V(p,x)\frac{\cos(\omega_i, n_p)\cos(-\omega_i, n_x)}{\pi r_{px}^2} \geq 0$
- Radiosity equation $B(p) = B_e(p) + \rho(p)\int_S K(p,x)B(x)dA_x$
- Kernel weights the contribution of patch x for the radiosity at patch p and vice versa



$$\theta_p = \cos(\omega_i, n_p)$$
$$\theta_x = \cos(-\omega_i, n_x)$$

K(p,x) gets larger
- if p and x are oriented towards each other
- if p and x are closer to each other

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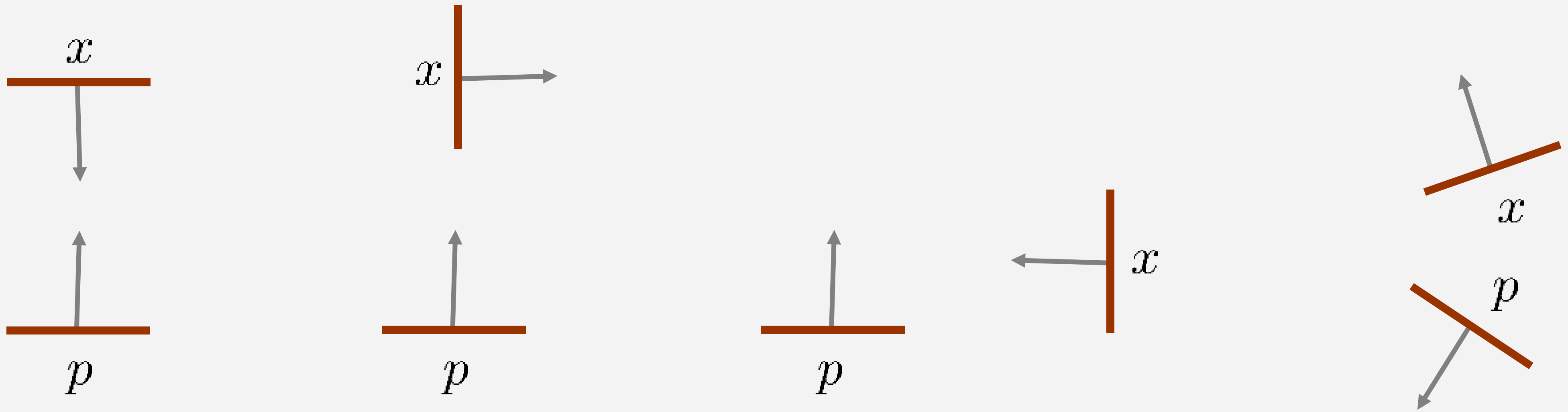
Kernel

– Indicates the “importance” of patch x for patch p



Large K(p,x) Small K(p,x) Small K(p,x) K(p,x) = 0

Discretization

- Continuous form of the radiosity equation
  - $B(p) = B_e(p) + \rho(p) \int_S K(p,x) B(x) dA_x$ Radiosity at points
  - Infinite number of equations for infinite number of unknowns
- Discretization (Finite Element Method)
  - $$\begin{pmatrix} 1-\rho_1 K_{11} A_1 & -\rho_1 K_{12} A_2 & \dots & -\rho_1 K_{1n} A_n \\ -\rho_2 K_{21} A_1 & 1-\rho_2 K_{22} A_2 & \dots & -\rho_1 K_{2n} A_n \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n K_{n1} A_1 & \dots & \dots & 1-\rho_n K_{nn} A_n \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} = \begin{pmatrix} B_{e1} \\ B_{e2} \\ \vdots \\ B_{en} \end{pmatrix}$$ Radiosity at faces
  - N equations for n unknowns
  - One unknown per face / triangle / finite element

From Differential Areas to Finite Areas

- Start with, e.g., a triangulated scene representation
- Assume constant radiosity over area A_i : $B(x_i) = \text{const} = B_i$
- Assume constant reflectance over area A_i : $\rho(x_i) = \text{const} = \rho_i$
- Integrate radiosity over a face i with area A_i
- Radiosity equation for face i

$$\int_{S_i} B(x_i) dA_{x_i} = \int_{S_i} B_e(x_i) dA_{x_i} + \int_{S_i} \rho(x_i) \int_S K(x_i, x) B(x) dA_x dA_{x_i}$$

$$A_i B_i = A_i B_{ei} + \rho_i \int_{S_i} \int_S K(x_i, x) B(x) dA_x dA_{x_i}$$

- B_i, B_{ei} are reflected and emitted radiosity per face i

From Differential Areas to Finite Areas

- $\int_S K(x_i, x) B(x) dA_x$ is an integral over all faces of a scene
- Can be written as $\sum_j \int_{S_j} K(x_i, x_j) B(x_j) dA_{x_j}$
  - Integral over a face j, summed over all faces
- Radiosity equation

$$A_i B_i = A_i B_{ei} + \rho_i \int_{S_i} \int_S K(x_i, x) B(x) dA_x dA_{x_i}$$

$$B_i = B_{ei} + \rho_i \frac{1}{A_i} \int_{S_i} \sum_j \int_{S_j} K(x_i, x_j) B(x_j) dA_{x_j} dA_{x_i}$$

Division by A_i

$$B_i = B_{ei} + \rho_i \frac{1}{A_i} \sum_j \int_{S_i} \int_{S_j} K(x_i, x_j) B(x_j) dA_{x_j} dA_{x_i}$$

From Differential Areas to Finite Areas

- $B_i = B_{ei} + \rho_i \frac{1}{A_i} \sum_j \int_{S_i} \int_{S_j} K(x_i, x_j) B(x_j) dA_{x_j} dA_{x_i}$
- Constant radiosity over area A_j, i.e. $B(x_j) = \text{const} = B_j$

  $B_i = B_{ei} + \rho_i \frac{1}{A_i} B_j \sum_j \int_{S_i} \int_{S_j} K(x_i, x_j) dA_{x_j} dA_{x_i}$
- Form factor: $F_{ij} = \frac{1}{A_i} \int_{S_i} \int_{S_j} K(x_i, x_j) dA_{x_j} dA_{x_i}$
- Almost discretized radiosity equation

  $B_i = B_{ei} + \sum_j \rho_i F_{ij} B_j$

Form Factor – A First Approximation

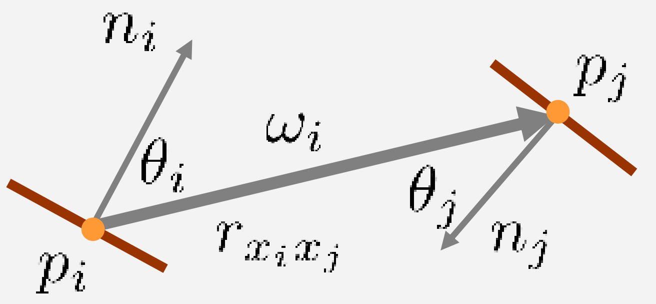
- $F_{ij} = \frac{1}{A_i} \int_{S_i} \int_{S_j} K(x_i, x_j) dA_{x_j} dA_{x_i}$
- Assume constant kernel for two faces i and j:

  $K(x_i, x_j) = \text{const} = K_{ij}$ Bad for pairs of faces that partially see each other

- $F_{ij} = \frac{1}{A_i} K_{ij} \int_{S_i} \int_{S_j} dA_{x_j} dA_{x_i} = K_{ij} A_j$
- Choose representative positions p_i, p_j on faces i and j

  $F_{ij} = V(p_i, p_j) \frac{\cos(\omega_i, n_i) \cos(-\omega_i, n_j)}{\pi r^2_{p_i p_j}} A_j$

  Non-zero for faces that "see" each other



$\theta_i = \cos(\omega_i, n_i)$

$\theta_j = \cos(-\omega_i, n_j)$

Discretization of the Radiosity Equation

– Continuous form, per surface position

$$B(p) = B_e(p) + \frac{\rho(p)}{\pi} \int_S B(x) V(p,x) \frac{\cos(\omega_i, n_p)\cos(-\omega_i, n_x)}{r_{px}^2} dA_x$$

– Discretized form, per face / triangle

Finite Element Method

$$B_i = B_{ei} + \sum_j \rho_i F_{ij} B_j$$

$$B_i - \sum_j \rho_i F_{ij} B_j = B_{ei}$$

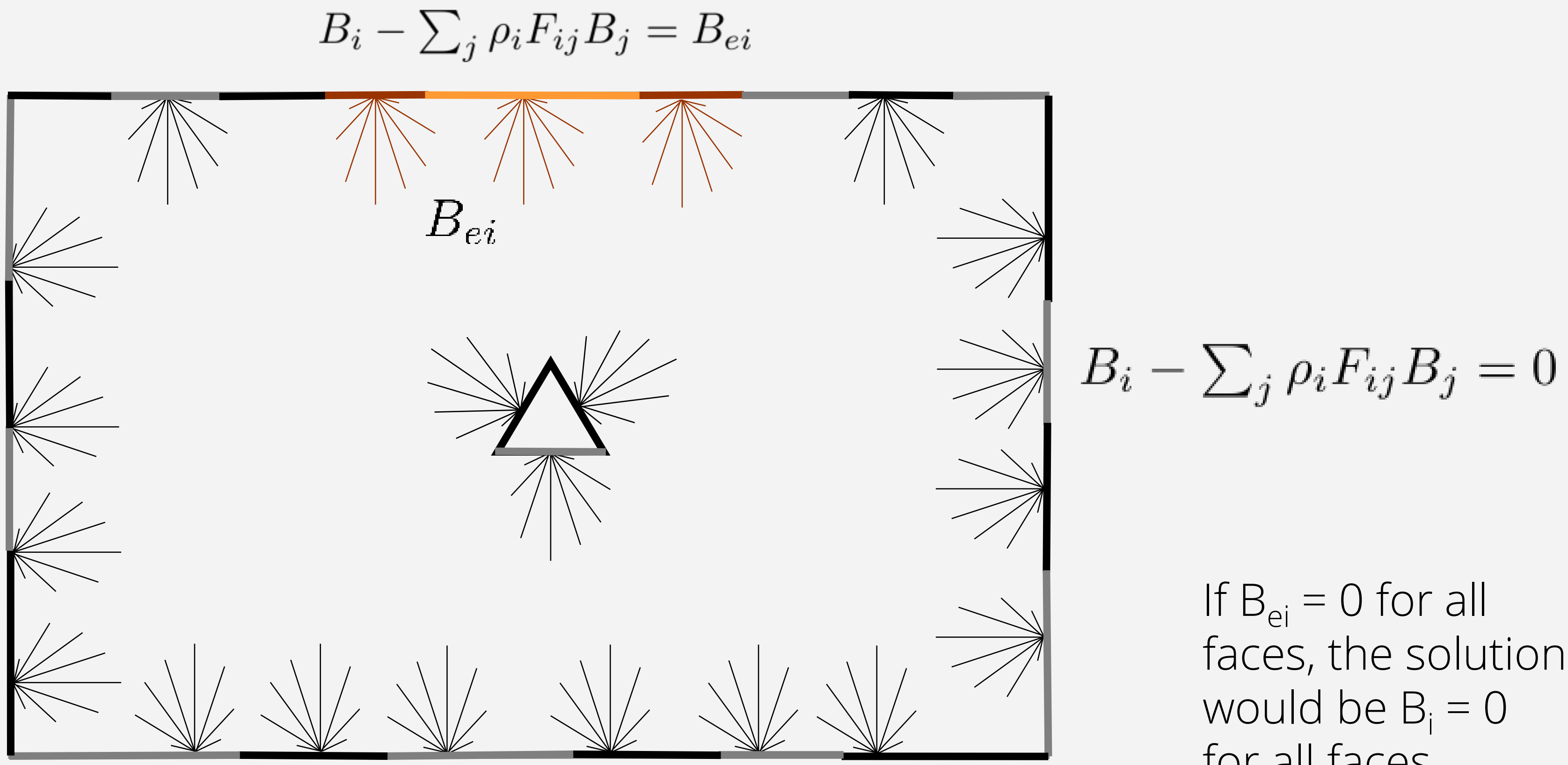
– B_{ei} is a source, i.e. the emitted radiosity at face i

– B_i, B_j are unknown radiosities at faces i and j

– ρ_i, F_{ij} are known coefficients

System of Linear Equations

– N equations for n unknown radiosities at n faces



If $B_{ei} = 0$ for all faces, the solution would be $B_i = 0$ for all faces.

System of Linear Equations

– $$\begin{pmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \dots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \dots & -\rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & \dots & -\rho_n F_{nn-1} & 1-\rho_n F_{nn} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} = \begin{pmatrix} B_{e1} \\ B_{e2} \\ \vdots \\ B_{en} \end{pmatrix}$$

Matrix with known coefficients, reflectances and form factors. Indirect illumiation. Describes, how faces illuminate each other.

Unknown radiosities.

Known source terms. Direct illumi-nation.

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Solving the Linear System

- Typically with iterative schemes, e.g. relaxed Jacobi
  - Initialize, e.g., $B_i^0 = 0$ Superscript indicates solver iteration
  - Iteratively update $B_i^{l+1} = B_i^l + \frac{\lambda_i}{1-\rho_i F_{ii}}(B_{ei} - (B_i^l - \sum_j \rho_i F_{ij} B_j^l))$

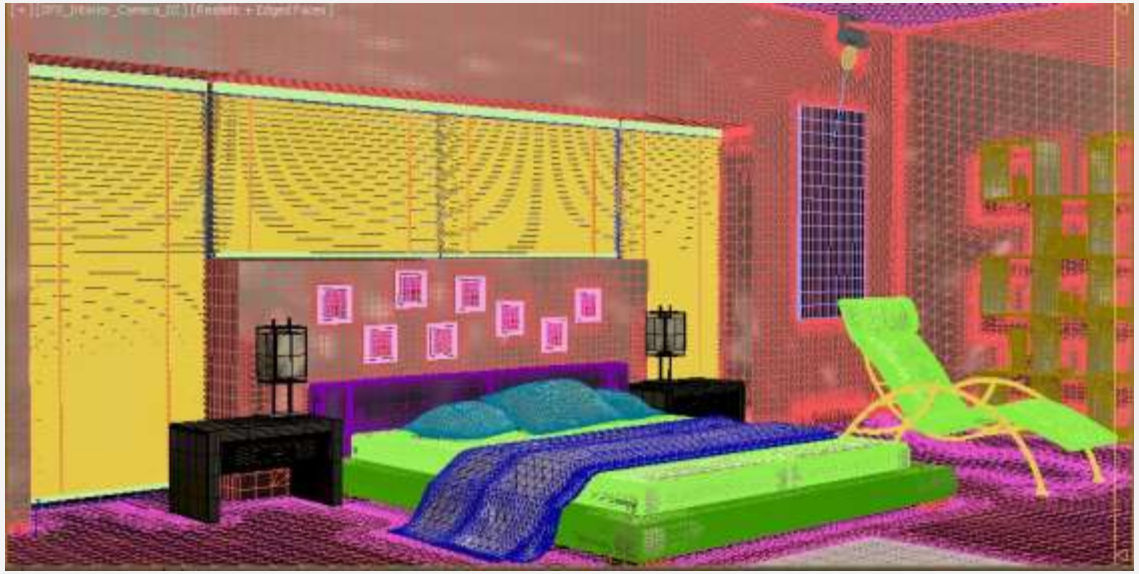
    λ_i is a user-defined parameter that governs the solver convergence
- Intuition
  - Changes from B_i^l to B_i^{l+1} are proportional to $B_{ei} - (B_i^l - \sum_j \rho_i F_{ij} B_j^l)$
  - If $B_{ei} - (B_i^l - \sum_j \rho_i F_{ij} B_j^l) = 0$, i.e. $B_i^l - \sum_j \rho_i F_{ij} B_j^l = B_{ei}$, the solver has converged and $B_i^{l+1} = B_i^l$

Summary

- Scene modeling / meshing
- Computation of form factors
- Solve linear system
- Set up a camera
- Project scene onto view plane / cast rays into the scene
- Lookup radiosity / reconstruct radiance per pixel

Meshing Example

Low / adaptive resolution



High resolution

[Aayush Chopra]

Rendering of the Solution

Final rendering from an arbitrary position and orientation.

[Aayush Chopra]

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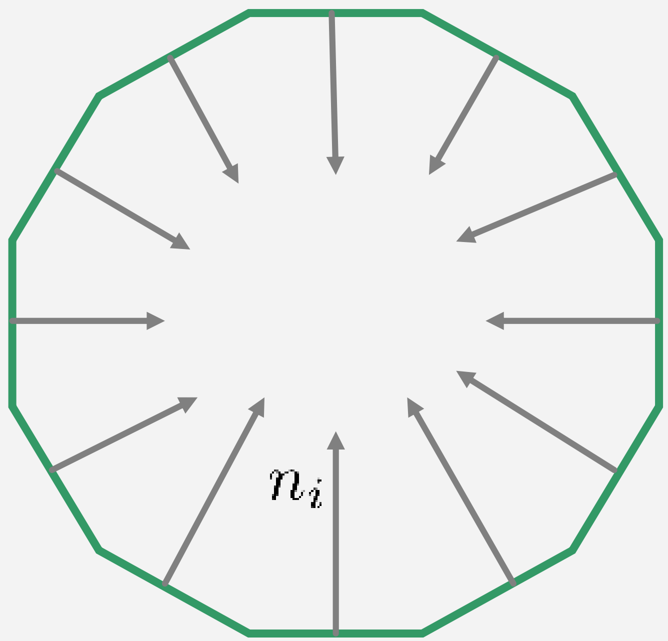
Form Factor Computation

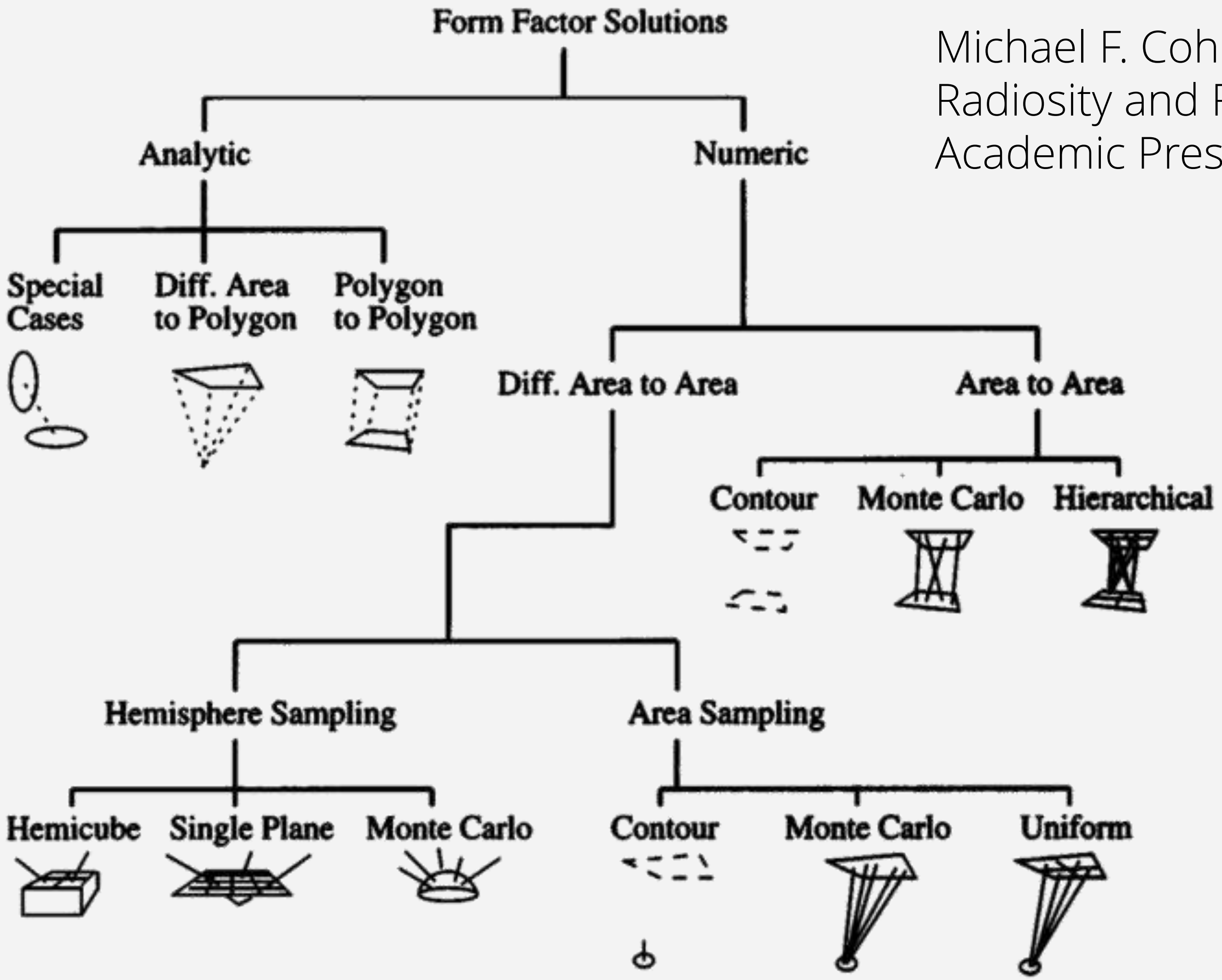
- Important and expensive
- Worst case
  - Inside of a convex polygon
  - All faces see each other
  - Complexity of a naive form factor computation is quadratic in the number of faces
  - System matrix is fully filled with non-zero entries



$$\forall i, \forall j : F_{ij} \neq 0$$

Form Factor Solutions

– Examples



Michael F. Cohen, John R. Wallace: Radiosity and Realistic Image Synthesis. Academic Press Professional, Boston.

Form Factor Properties

- Positive

$$F_{ij} = \frac{1}{A_i} \int_{S_i} \int_{S_j} V(x_i, x_j) \frac{\cos(\omega_i, n_i) \cos(-\omega_i, n_j)}{\pi r^2_{x_i x_j}} dA_{x_j} dA_{x_i} \geq 0$$

- Reciprocity relation

$$K(x_i, x_j) = V(x_i, x_j) \frac{\cos(\omega_i, n_i) \cos(-\omega_i, n_j)}{\pi r^2_{x_i x_j}} = K(x_j, x_i)$$

$$\begin{aligned} A_i F_{ij} &= A_i \frac{1}{A_i} \int_{S_i} \int_{S_j} K(x_i, x_j) dA_{x_j} dA_{x_i} \\ &= A_j \frac{1}{A_j} \int_{S_j} \int_{S_i} K(x_j, x_i) dA_{x_i} dA_{x_j} \\ &= A_j F_{ji} \end{aligned}$$

Form Factor Properties

- $\sum_j F_{ij} = \frac{1}{A_i} \int_{S_i} \sum_j \int_{S_j} V(x_i, x_j) \frac{\cos(\omega_i, n_i)\cos(-\omega_i, n_j)}{\pi r^2_{x_i x_j}} dA_{x_j} dA_{x_i}$

  $= \frac{1}{A_i} \int_{S_i} \int_{2\pi} \frac{\cos(\omega_i, n_i)}{\pi} d\omega_i dA_{x_i}$

  $= \frac{1}{A_i} \int_{S_i} \frac{\pi}{\pi} dA_{x_i} = 1$

- $\sum_j F_{ij} = 1 \Rightarrow \rho_i \sum_j F_{ij} \leq 1 \Rightarrow \rho_i \sum_{j \neq i} F_{ij} \leq 1 - \rho_i F_{ii}$
- Important for the convergence of iterative solvers
  - Diagonally dominant system matrix
  - Sum of magnitudes of non-diagonal entries per row is smaller than the magnitude of the diagonal entry
  - Surface properties influence convergence

System Notation

- $$\begin{pmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \dots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \dots & -\rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & \dots & -\rho_n F_{nn-1} & 1-\rho_n F_{nn} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} = \begin{pmatrix} B_{e1} \\ B_{e2} \\ \vdots \\ B_{en} \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} \rho_1 F_{11} & \rho_1 F_{12} & \dots & \rho_1 F_{1n} \\ \rho_2 F_{21} & \rho_2 F_{22} & \dots & \rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_n F_{n1} & \dots & \rho_n F_{nn-1} & \rho_n F_{nn} \end{pmatrix}$$

- $(\mathbf{I}-\mathbf{F})\mathbf{B} = \mathbf{B_e} \qquad \mathbf{B} = \mathbf{B_e} + \mathbf{FB}$

System Interpretation

- Radiosity equation per point
  - $B(p) = B_e(p) + \int_S f_r(p, \omega_i \leftrightarrow \omega_o) B(x) V(p,x) \frac{\cos(\omega_i, n_p)\cos(-\omega_i, n_x)}{r_{px}^2} dA_x$
- System of per-face discretized radiosity equations
  - $\mathbf{B} = \mathbf{B_e} + \mathbf{FB}$
  - $\mathbf{B}$ overall radiosity at all faces
  - $\mathbf{B_e}$ radiosity at all faces due to emission
  - $\mathbf{FB}$ radiosity at all faces due to the reflection of incident radiosity $\mathbf{B}$ from all faces

System Solution

- $(\mathbf{I}-\mathbf{F})\mathbf{B} = \mathbf{B_e}$

  $\mathbf{B} = (\mathbf{I}-\mathbf{F})^{-1}\mathbf{B_e}$

- Neumann series

  $(\mathbf{I}-\mathbf{F})^{-1} = \sum_{k=0}^{\infty} \mathbf{F}^k$

  The inverse does not always exist. In particular, there is no solution for unphysical settings.

  $\mathbf{B} = \mathbf{B_e} + \mathbf{F}\mathbf{B_e} + \mathbf{F}\mathbf{F}\mathbf{B_e} + \mathbf{F}\mathbf{F}\mathbf{F}\mathbf{B_e} + \ldots$

Terms in the Neumann Series

- $\mathbf{B} = \mathbf{B_e} + \mathbf{F}\mathbf{B_e} + \mathbf{F}\mathbf{F}\mathbf{B_e} + \mathbf{F}\mathbf{F}\mathbf{F}\mathbf{B_e} + \ldots$
  - $\mathbf{B_e}$ emitted radiosity
  - $\mathbf{F}\mathbf{B_e}$ reflected radiosity due to incident emitted radiosity $\mathbf{B_e}$ (emitted radiosity after one bounce at a surface)
  - $\mathbf{F}\mathbf{F}\mathbf{B_e}$ reflected radiosity due to incident radiosity that was reflected due to incident emitted radiosity $\mathbf{B_e}$ (emitted radiosity after two bounces at surfaces)
  - $\mathbf{F}\mathbf{F}\mathbf{F}\mathbf{B_e}$ ...

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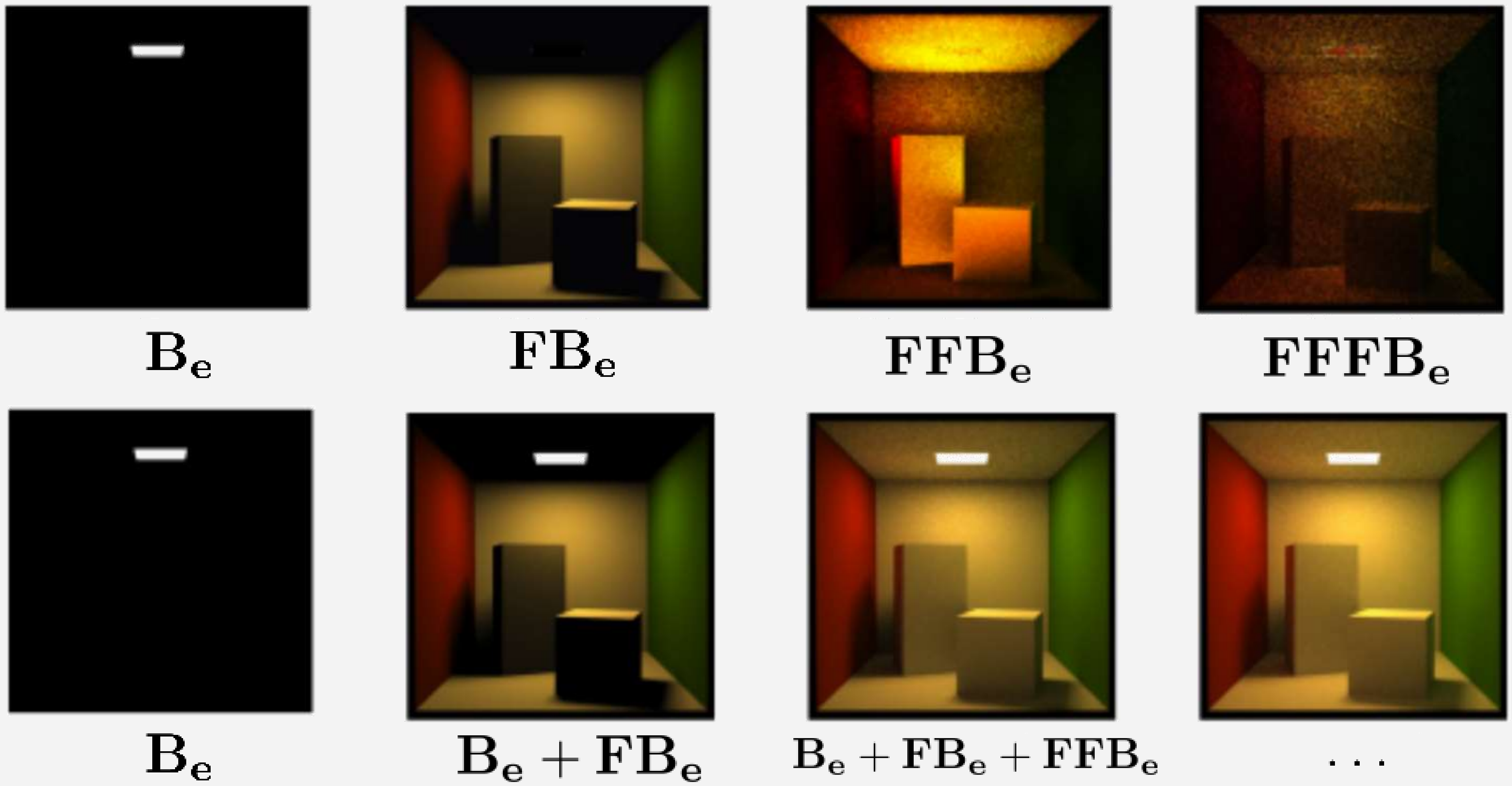
Terms in the Neumann Series

- $\mathbf{B} = \mathbf{B_e} + \mathbf{F B_e} + \mathbf{F F B_e} + \mathbf{F F F B_e} + \ldots$
  - $\mathbf{B_e}$ contribution of emitted light to the solution
  - $\mathbf{F B_e}$ contribution of emitted light after one bounce
  - $\mathbf{F F B_e}$ contribution of emitted light after two bounces
  - $\mathbf{F F F B_e}$ contribution of emitted light after three bounces

Visualizing the Neumann Series



[Pat Hanrahan]

Solver

- Jacobi with, e.g., $\frac{\lambda_i}{1-\rho_i F_{ii}} = 1$: $B_i^{l+1} = B_{ei} + \sum_j \rho_i F_{ij} B_j^l$
  - $\mathbf{B}^{l+1} = \mathbf{B_e} + \mathbf{F}\mathbf{B}^l$
- Iterations
  - $\mathbf{B}^0 = \mathbf{0}$
  - $\mathbf{B}^1 = \mathbf{B_e}$
  - $\mathbf{B}^2 = \mathbf{B_e} + \mathbf{F}\mathbf{B}^1 = \mathbf{B_e} + \mathbf{F}\mathbf{B_e}$
  - $\mathbf{B}^3 = \mathbf{B_e} + \mathbf{F}\mathbf{B}^2 = \mathbf{B_e} + \mathbf{F}\left(\mathbf{B_e} + \mathbf{F}\mathbf{B_e}\right)$
    $= \mathbf{B_e} + \mathbf{F}\mathbf{B_e} + \mathbf{F}\mathbf{F}\mathbf{B_e}$
- Intuition does not necessarily apply to other solvers

Solver Convergence

- Radiosity changes $\mathbf{B_e}$ $\mathbf{FB_e}$ $\mathbf{FFB_e}$ $\mathbf{FFFB_e}$ should get “smaller” with each iteration
  - Spectral radius of $\mathbf{F}$ should be smaller one
  - Some faces in a scene should partially absorb light
  - Faces should not “generate” light, i.e. $\rho_i > 1$
  - $$\mathbf{F} = \begin{pmatrix} \rho_1 F_{11} & \rho_1 F_{12} & \dots & \rho_1 F_{1n} \\ \rho_2 F_{21} & \rho_2 F_{22} & \dots & \rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_n F_{n1} & \dots & \rho_n F_{nn-1} & \rho_n F_{nn} \end{pmatrix} \quad \begin{array}{l} 0 \le \rho_i \le 1 \\ \sum_j F_{ij} = 1 \end{array}$$

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