

Image Processing and Computer Graphics

Lighting

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Motivation

- modeling of visual phenomena
 - light is emitted by light sources, e.g. sun or lamp
 - light interacts with objects
 - light is absorbed or scattered (reflected) at surfaces
 - light is absorbed by a sensor, e.g. human eye or camera



[Akenine-Möller et al.]

Outline

- light
- color
- lighting models
- shading models

Light

- modeled as
 - electromagnetic waves, radiation
 - photons
 - geometric rays
- photons
 - particles
 - characterized by wavelength
(perceived as color in the visible spectrum)
 - carry energy
(inversely proportional to the wavelength)
 - travel along a straight line at the speed of light

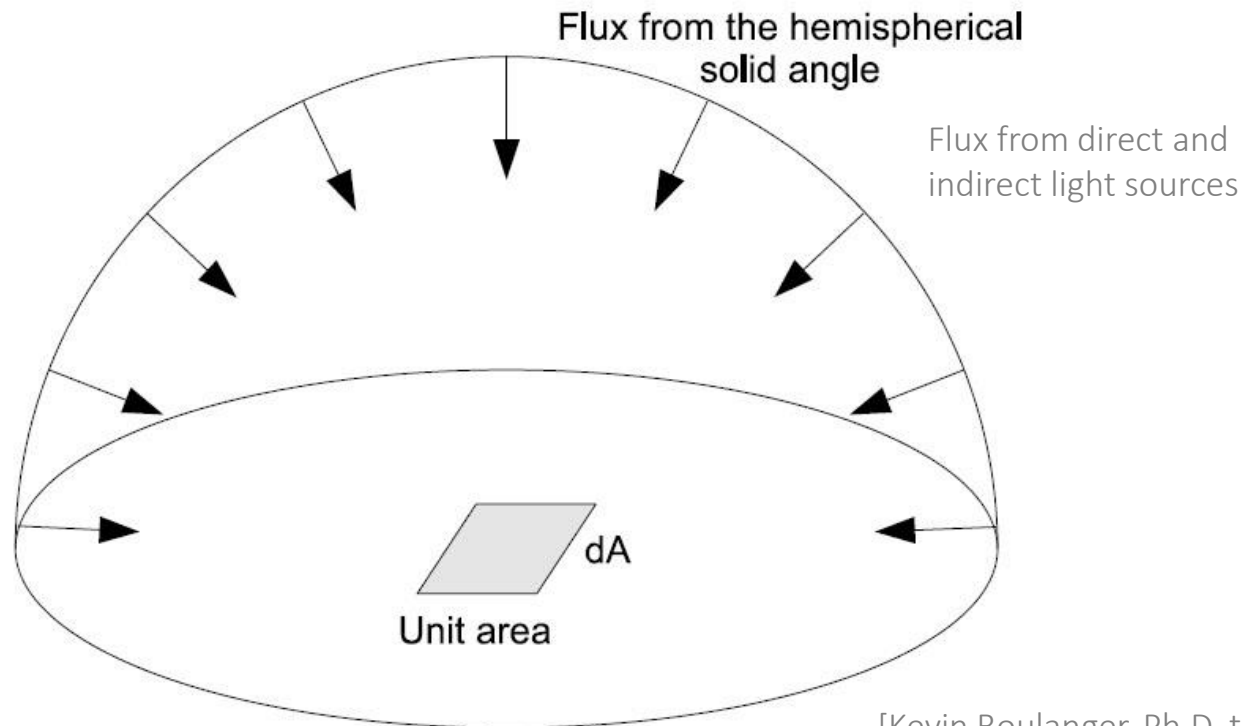
Radiometric Quantities

- radiant energy Q
 - photons have some radiant energy
- radiant flux Φ , radiant power P
 - rate of flow of radiant energy per unit time: $\Phi = \frac{dQ}{dt}$
 - e.g., overall energy of photons emitted by a source per time
- flux density (irradiance, radiant exitance)
 - radiant flux per unit area: $E = \frac{d\Phi}{dA}$
 - rate at which radiation is incident on,
or exiting a flat surface area dA
 - describes strength of radiation with respect to a surface area
 - no directional information

Radiometric Quantities

Irradiance

- irradiance measures the overall radiant flux Φ (light flow, photons per unit time) into a surface element

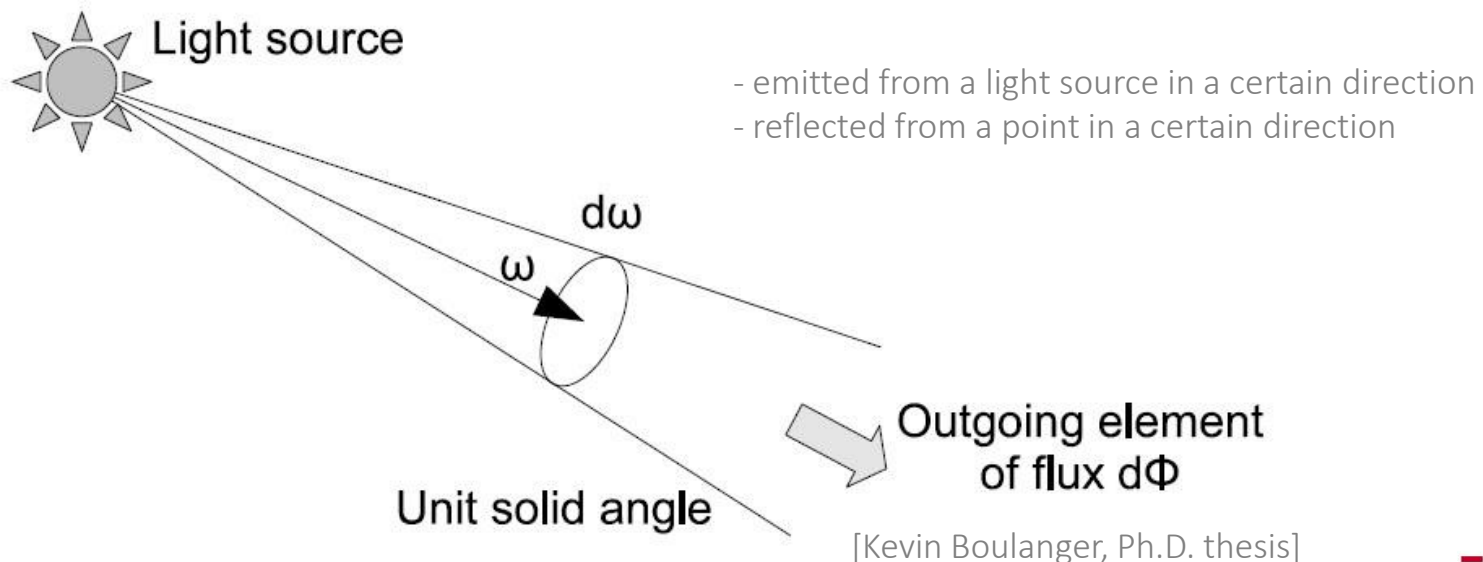


[Kevin Boulanger, Ph.D. thesis]

Radiometric Quantities

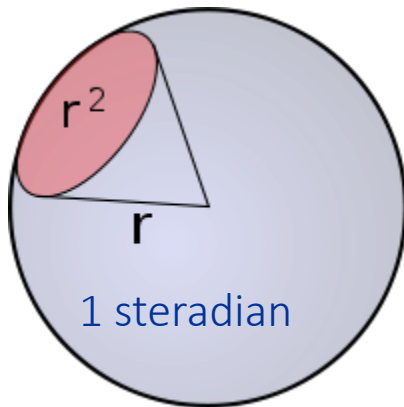
Radiant Intensity

- radiant intensity
 - radiant flux per unit solid angle: $I = \frac{d\Phi}{d\omega}$
 - radiant flux Φ (light flow, photons per unit time) incident on, emerging from, or passing through a point in a certain direction



Solid Angle

- 2D angle in 3D space
- measured in steradians
 - A steradian is the solid angle subtended at the center of a sphere of radius r by a portion of the sphere surface with area $A=r^2$.



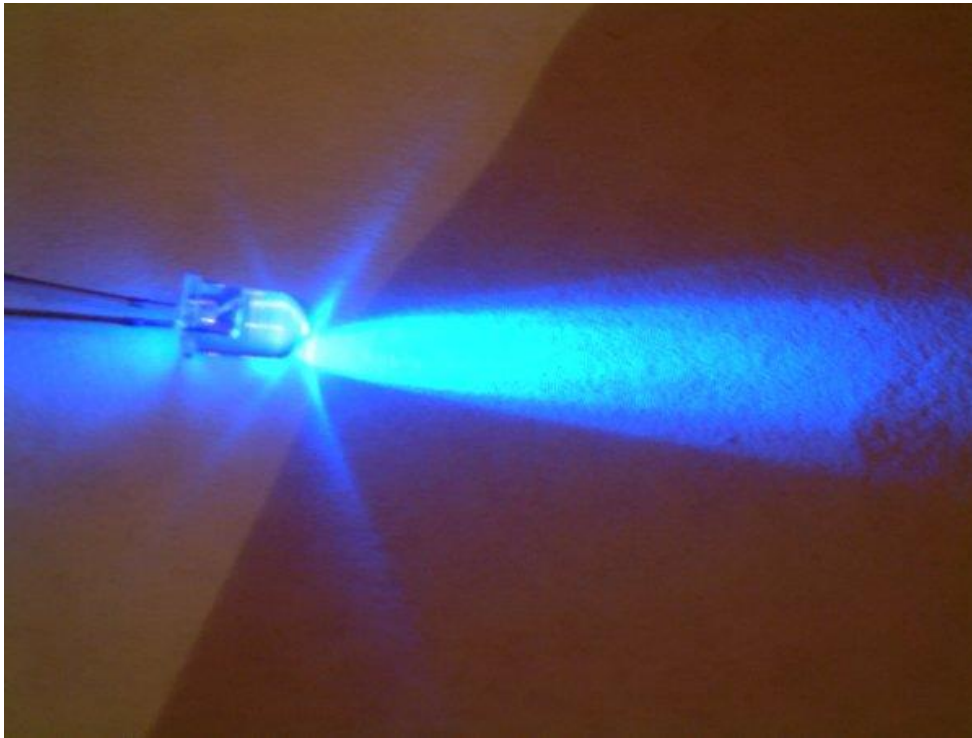
$$d\omega = \frac{dA}{r^2}$$

[Wikipedia: Solid angle]

Radiometric Quantities

Radiant Intensity

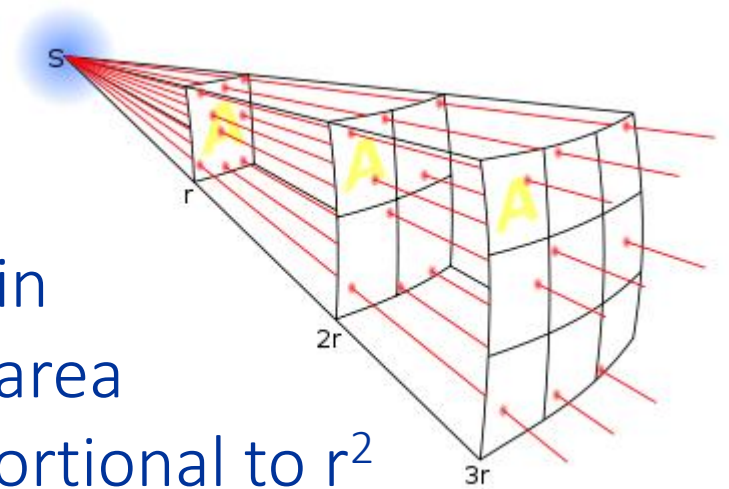
- light source with direction-dependent radiant intensities



[Wikipedia: Strahldichte]

Inverse Square Law

- point light source with radiant intensity I in direction ω
- irradiance along a ray in direction ω is inversely proportional to the square of the distance r from the source $E = \frac{I}{r^2}$
- the number of photons emitted in direction $d\omega$ and hitting surface area dA at distance r is inversely proportional to r^2
- the area subtended by a solid angle is proportional to r^2
- $$E = \frac{d\Phi}{dA} = \frac{d\Phi}{r^2 d\omega} = \frac{I}{r^2}$$



[Wikipedia: Inverse Square Law]

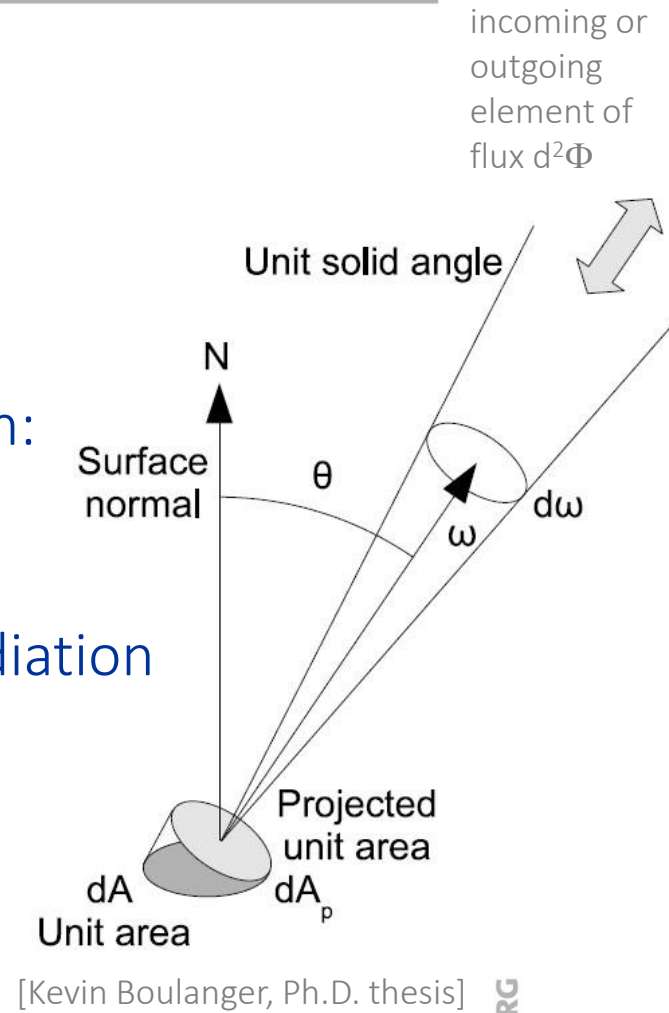
Radiometric Quantities

Radiance

- radiance
 - radiant flux per unit solid angle per unit projected area incident on, emerging from, passing through a surface element in a certain direction:

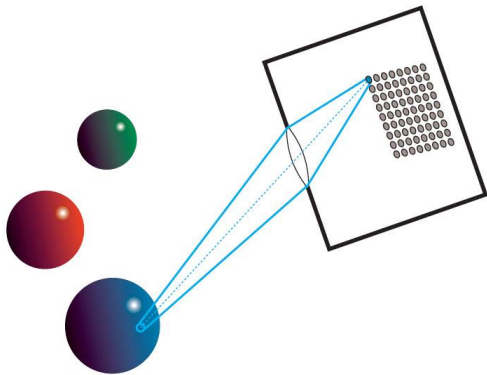
$$L = \frac{d^2\Phi}{dA_p d\omega} = \frac{d^2\Phi}{dA \cos \theta d\omega}$$

- describes strength and direction of radiation
- constant radiance in all directions corresponds to a radiant intensity that is proportional to $\cos \theta$, but constant radiant intensity results in different radiance

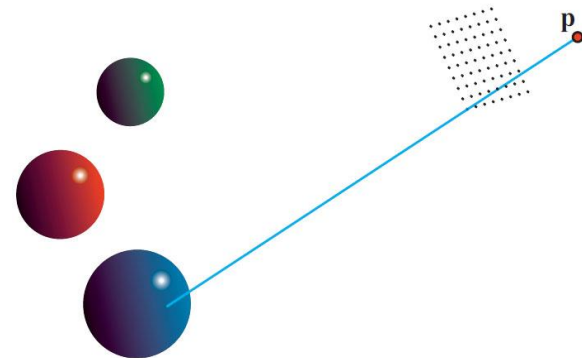


Radiance and Sensors

- radiance
 - is measured by sensors
 - is computed in computer-generated images
 - is preserved along lines in space
 - does not change with distance



a sensor with a small area receives light from a small set of directions, i. e. radiance



idealized graphics model of an imaging sensor

[Akenine-Möller et al.]

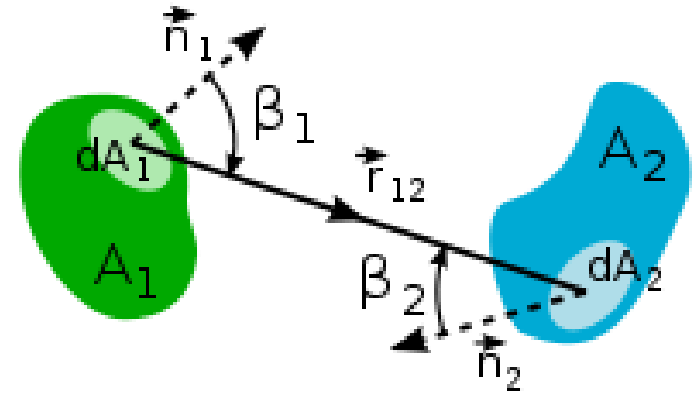
Conservation of Radiance

- outgoing flux from dA_1 into direction r_{12}

$$d^2\Phi_1 = L_1 \cdot \cos \beta_1 \cdot dA_1 \cdot d\omega_1$$

$$d\omega_1 = \cos \beta_2 \cdot dA_2 \cdot r^{-2}$$

$$d^2\Phi_1 = \frac{L_1 \cdot \cos \beta_1 \cdot \cos \beta_2 \cdot dA_1 \cdot dA_2}{r^2}$$



- incoming flux to dA_2 from direction $-r_{12}$

$$d^2\Phi_2 = L_2 \cdot \cos \beta_2 \cdot dA_2 \cdot d\omega_2 = \frac{L_2 \cdot \cos \beta_1 \cdot \cos \beta_2 \cdot dA_1 \cdot dA_2}{r^2}$$

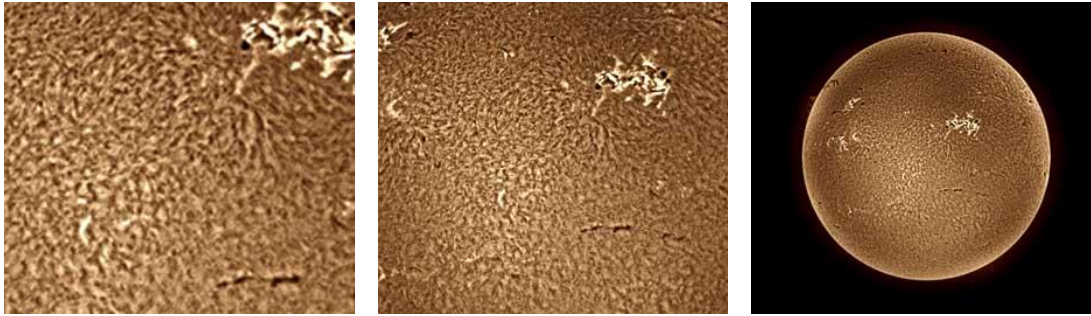
- if flux is preserved, the radiance does not change

$$d^2\Phi_1 = d^2\Phi_2 \Leftrightarrow L_1 = L_2$$

[Wikipedia: Strahldichte]

Radiance and Sensors

- three imaginary photos of the Sun from different distances



J. J. Condon and S. M. Ransom:

"Essential Radio Astronomy", National Radio Astronomy Observatory

- radiance in a pixel does not depend on the distance
 - irradiance of a pixel received from dA on the Sun is proportional to $1/r^2$
 - dA on the Sun captured by a pixel in directions $d\omega$ is proportional to r^2

Radiometric Quantities

radiometric quantity	symbol	unit	German
radiant energy	Q	J	Strahlungsenergie
radiant flux	Φ	W	Strahlungsfluss
flux density (irradiance, radiant exitance)	E	$\text{W} \cdot \text{m}^{-2}$	Strahlungsstromdichte
radiant intensity	I	$\text{W} \cdot \text{sr}^{-1}$	Strahlungsstärke / Strahlungsintensität
radiance	L	$\text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1}$	Strahldichte

Photometric / Radiometric Quantities

Fotometrische Größe	Einheit	Strahlungsgröße	Einheit
Lichtmenge	$\text{lm} \cdot \text{s}$	Strahlungsenergie	J
Lichtstrom	lm	Strahlungsfluss	W
Beleuchtungsstärke Lichtausstrahlung	lx $\text{lm} \cdot \text{m}^{-2}$	Bestrahlungsstärke Ausstrahlung	$\text{W} \cdot \text{m}^{-2}$
Lichtstärke	cd	Strahlungsstärke / Strahlungsintensität	$\text{W} \cdot \text{sr}^{-1}$
Leuchtdichte	$\text{cd} \cdot \text{m}^{-2}$	Strahldichte	$\text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1}$

Summary

- light consists of photons
- irradiance and radiant exitance describe the flux, i.e. the number of photons per time, into or from a surface per area
- radiant intensity is flux into a direction per solid angle
- radiance describes the flow into or from a surface from a certain direction per area per solid angle
- radiance is measured in sensors
- irradiance of a surface is inversely proportional to its squared distance for a point source with a certain radiant intensity
- radiance is preserved along straight (empty) lines

Outline

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Visible Spectrum

- photons are characterized by a wavelength within the visible spectrum from 390 nm to 750 nm
- light consists of a set of photons
- the distribution of wavelengths within this set is referred to as the spectrum of light
- spectra are perceived as colors

Visible Spectrum

- if the spectrum consists of a dominant wavelength, humans perceive a "rainbow" color (monochromatic)



[Wikipedia: Visible spectrum]

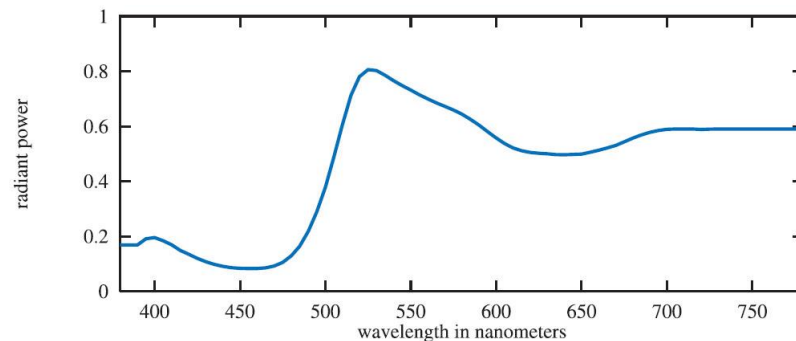
390 nm

750 nm

- equally distributed wavelengths are perceived as a shade of gray, ranging from black to white (achromatic)
- otherwise, colors "mixed from rainbow colors" are perceived (chromatic)

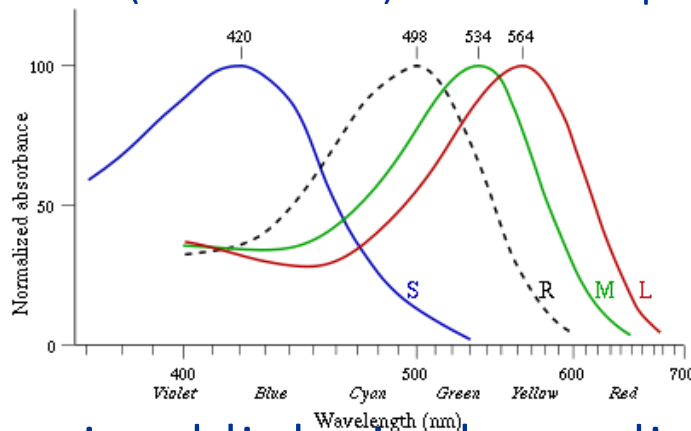
This spectrum corresponds to a ripe brown banana under white light

[Akenine-Möller et al.]



Human Eye

- is sensitive to radiation within the visible spectrum
- has sensors for day and night vision
 - three types of cones (Zapfen) for photopic (day) vision
 - rods (Stäbchen) for scotopic (night) vision



Normalised absorption spectra of human cone (S,M,L) and rod (R) cells. Cones are sensitive to blue, green, red.

[Wikipedia: Trichromacy]

- perceived light is the radiation spectrum weighted with absorption spectra (sensitivity) of the eye
- in daylight, three cone signals are interpreted by the brain

CIE Color Space

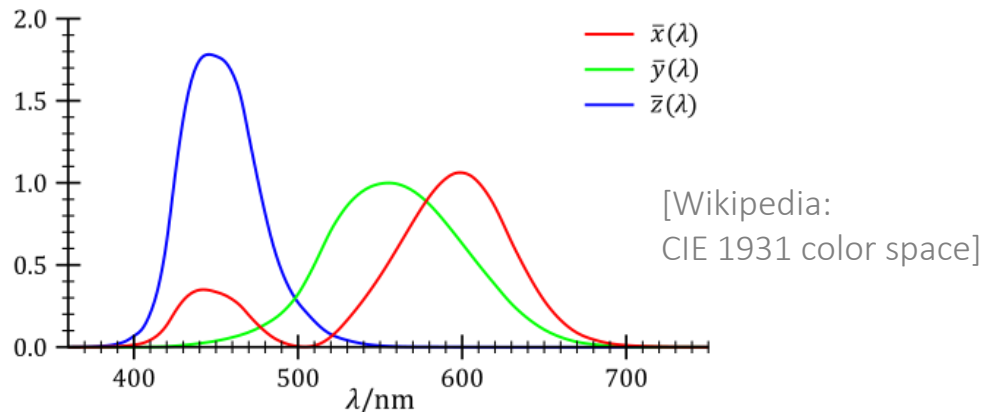
- XYZ color space

- created by the Int. Commission on Illumination CIE in 1931

$$X = \int_{\lambda} I(\lambda) \bar{x}(\lambda) d\lambda$$

$$Y = \int_{\lambda} I(\lambda) \bar{y}(\lambda) d\lambda$$

$$Z = \int_{\lambda} I(\lambda) \bar{z}(\lambda) d\lambda$$



- spectrum I is represented by X, Y, Z
given the color-matching functions $\bar{x}, \bar{y}, \bar{z}$
- the color-matching functions correspond to the spectral sensitivities of the cones of a standard observer

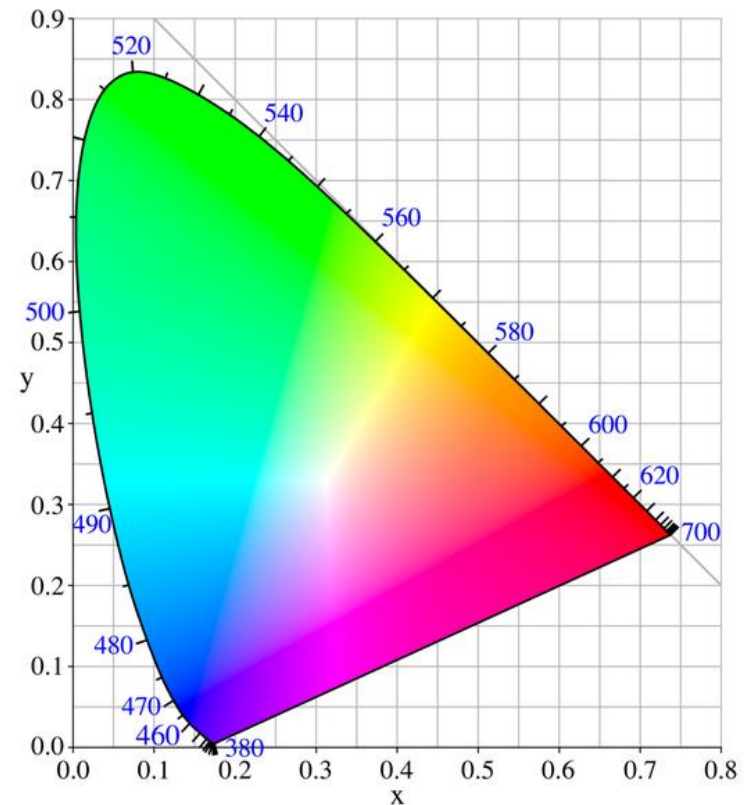
CIE xy Chromaticity Diagram

- XYZ represents color and brightness / luminance
- two values are sufficient to represent color

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

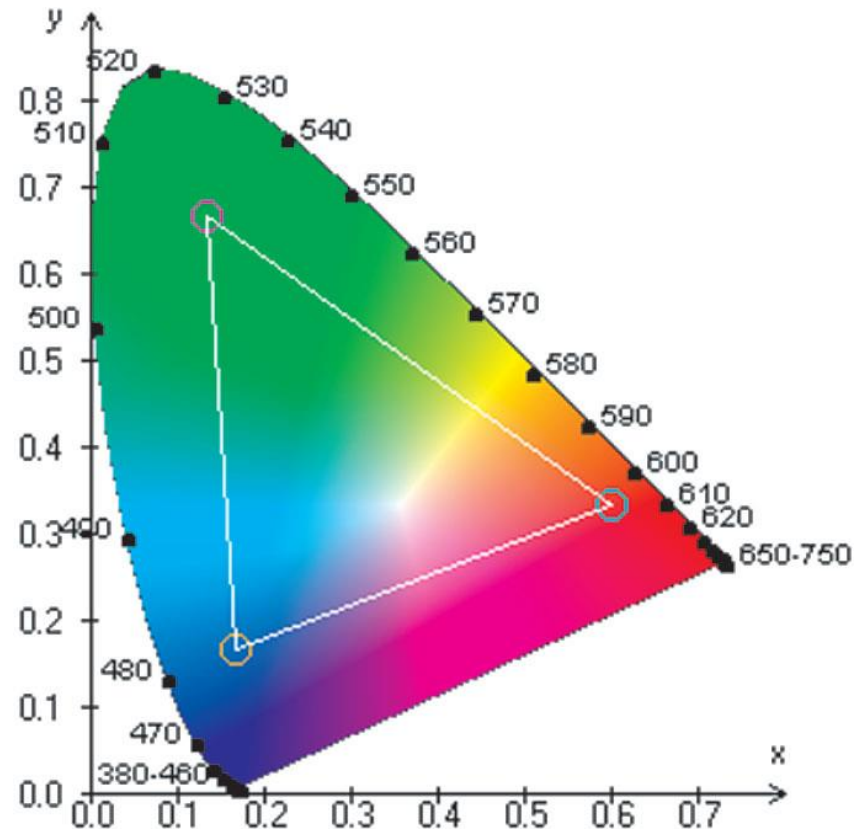
- monochromatic colors are on the boundary
- the center is achromatic



[Wikipedia: CIE 1931 color space]

Display Devices

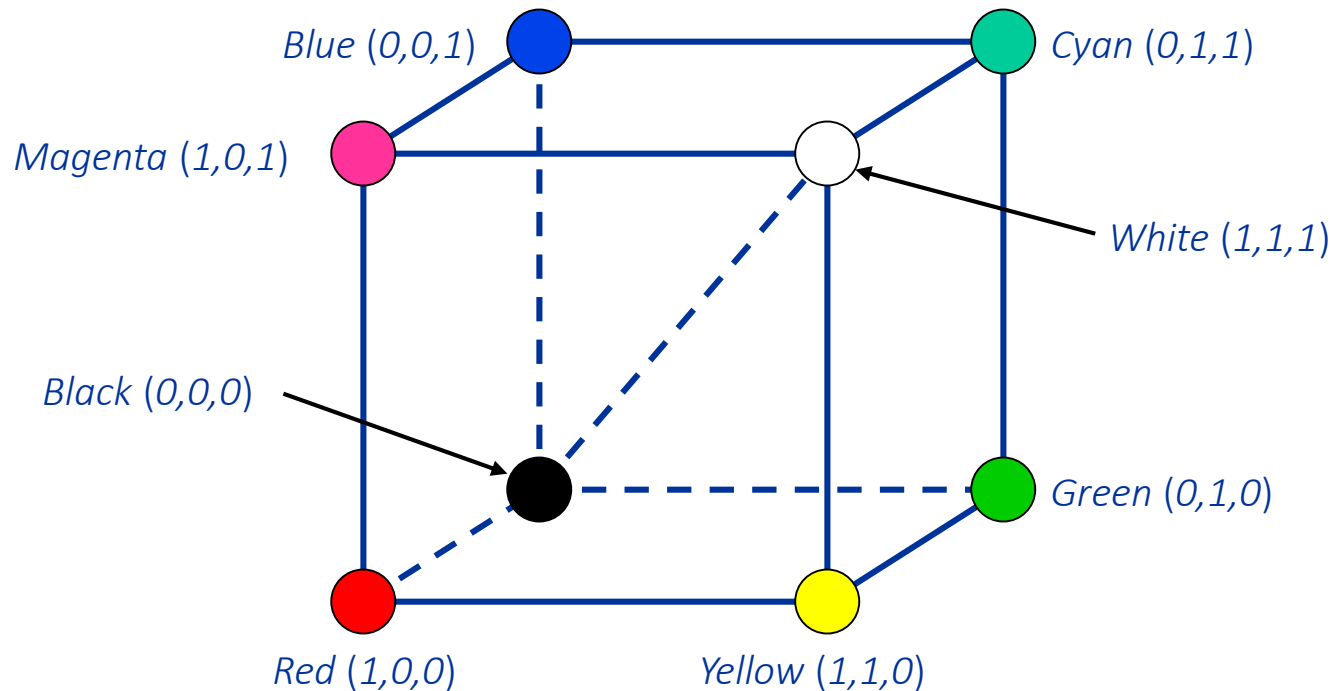
- use three primary colors
 - an example is indicated in the diagram in the gamut
- can only reproduce colors within the spanned triangle (gamut)
- → colors outside the gamut are not properly displayed on a monitor



[Akenine-Möller et al.]

RGB Color Space

- three primaries: red, green, blue



RGB Color Space

Lights and Objects

- light source color
 - e.g., yellow light (1, 1, 0)
 - emits a spectrum with maximum red and green components
 - the spectrum does not contain any blue
 - the RGB values describe the amount of the respective color component in the emitted light
- object color
 - e.g., yellow object (1, 1, 0)
 - perfectly reflects red and green comp. of the incoming light
 - perfectly absorbs the blue component of the incoming light
 - the RGB values describe how much of the respective incoming color component is reflected ("one minus value" describes how much is absorbed)

Summary

- the distribution of wavelengths within the flux of the perceived radiance is referred to as the spectrum of light
- spectra are weighted with absorption spectra of the eye and perceived as colors
- three cone types for daylight vision motivate XYZ space
- RGB space is often used for display devices
- colors of display devices are restricted to a gamut that does not contain all perceivable colors

Outline

- light
- color
- lighting models
- shading models

Motivation

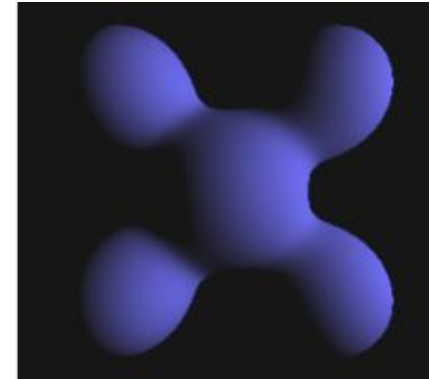
- compute the interaction of objects with light based on an illumination model (reflection model, lighting model)
- account for a variety of light sources and material properties
- but, keep it fast
 - only use **local information** per vertex / fragment
 - light source color, direction, and distance
 - object color (material) and surface normal
 - viewer direction
 - interaction between objects is neglected
 - interreflections among objects, occluded light sources or areal light sources are not handled

Outline

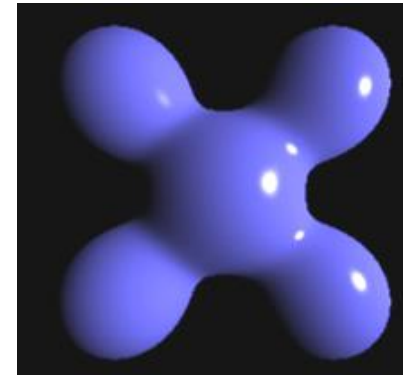
- light
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 - diffuse reflection
 - specular reflection
 - ambient light
 - Phong illumination model
 - miscellaneous
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Diffuse vs. Specular

- diffuse
 - incident light is reflected into many different directions
 - matte surfaces
- specular (mirror-like)
 - incident light is reflected into a dominant direction (perceived as small intense specular highlight)
 - shiny surfaces
- diffuse and specular reflection are **material properties**



ideal diffuse
reflecting surface

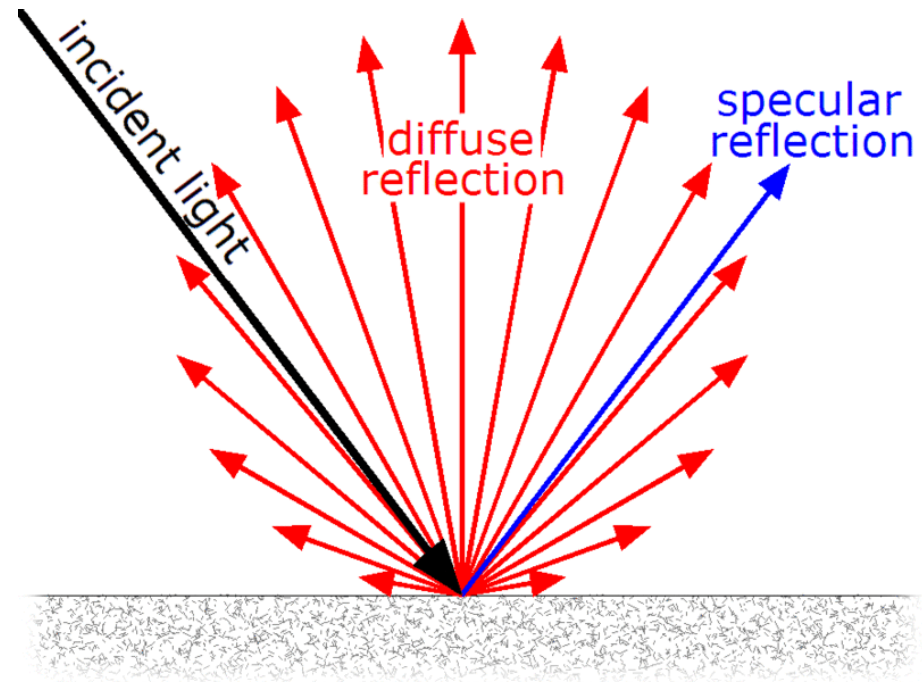


diffuse and specular
reflecting surface

[Wikipedia: Phong Shading]

Diffuse vs. Specular

- paper and marble have diffuse reflecting surfaces
- in general, materials are characterized by a combination of diffuse and specular reflection

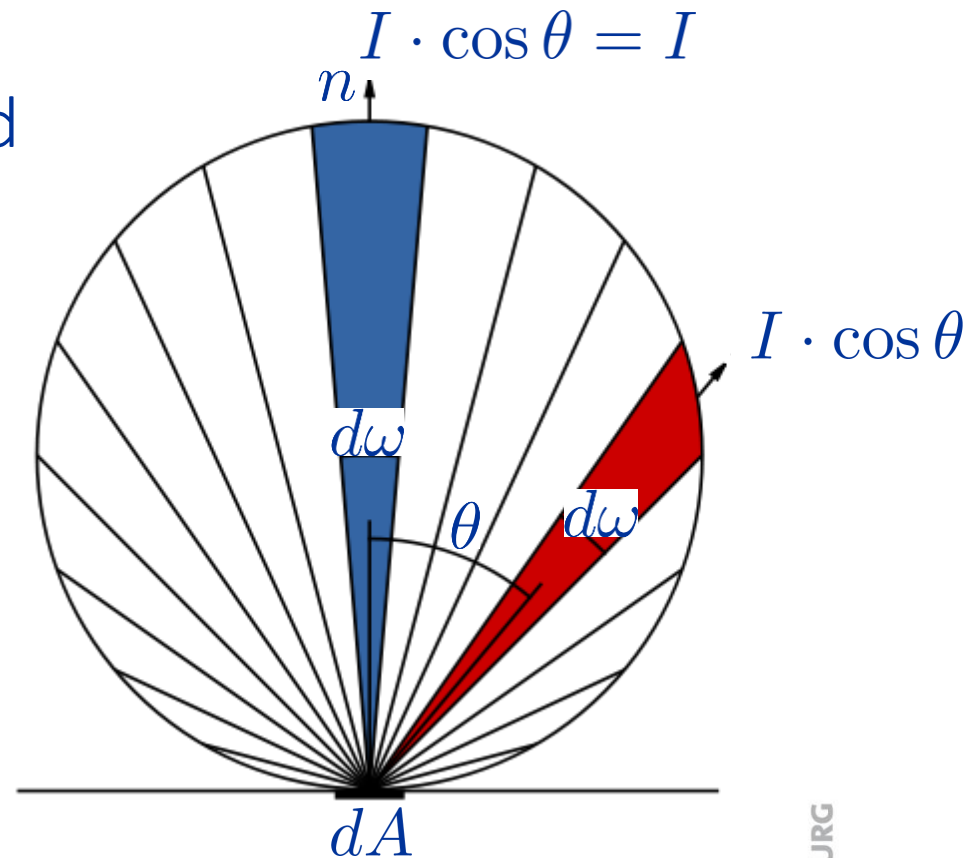


incident and reflected radiant flux of a diffuse and specular reflecting surface

[Wikipedia: Diffuse Reflection]

Lambert's Cosine Law

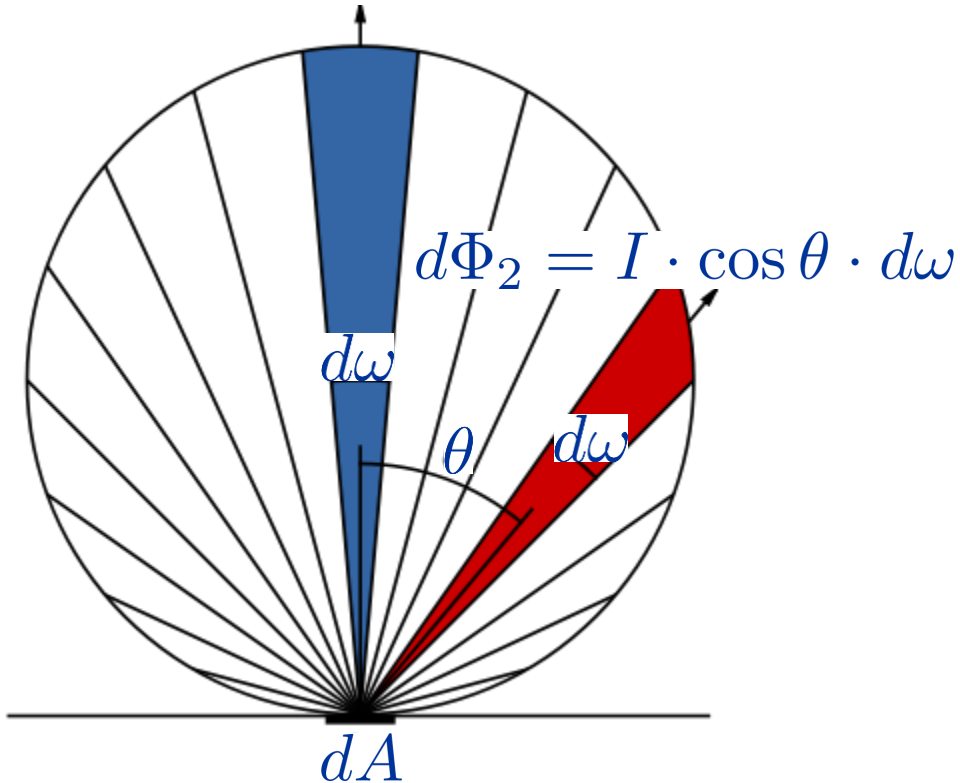
- computation of diffuse reflection is governed by Lambert's cosine law
- radiant intensity reflected from a "Lambertian" (matte, diffuse) surface is proportional to the cosine of the angle between view direction and surface normal n



[Wikipedia: Lambert's Cosine Law]

Lambert's Cosine Law

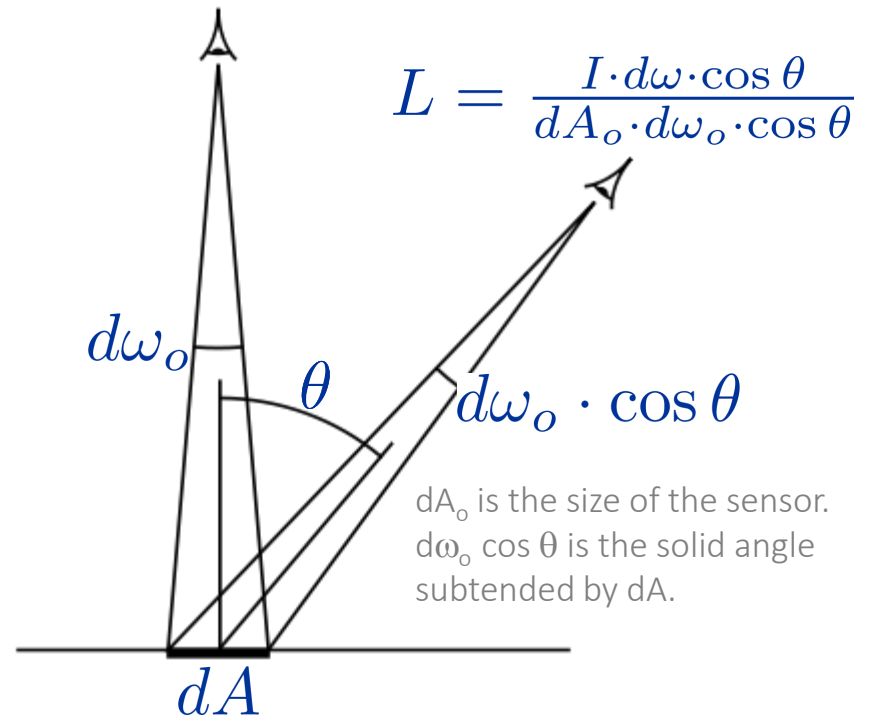
$$d\Phi_1 = I \cdot \cos 0 \cdot d\omega = I \cdot d\omega$$



Radiant intensity I and flux Φ are proportional to the cosine of the angle θ .

[Wikipedia: Lambert's Cosine Law]

$$L = \frac{I \cdot d\omega}{dA_o \cdot d\omega_o}$$

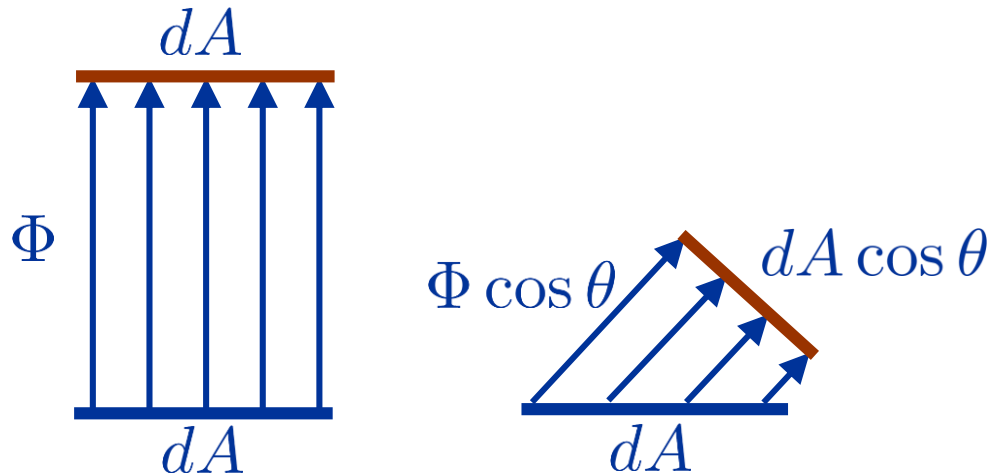


dA_o is the size of the sensor.
 $d\omega_o \cos \theta$ is the solid angle subtended by dA .

Radiance L measured at a sensor is independent from angle θ and distance.
 Computation of diffuse reflection is independent from the viewer direction and distance.

Lambert's Cosine Law

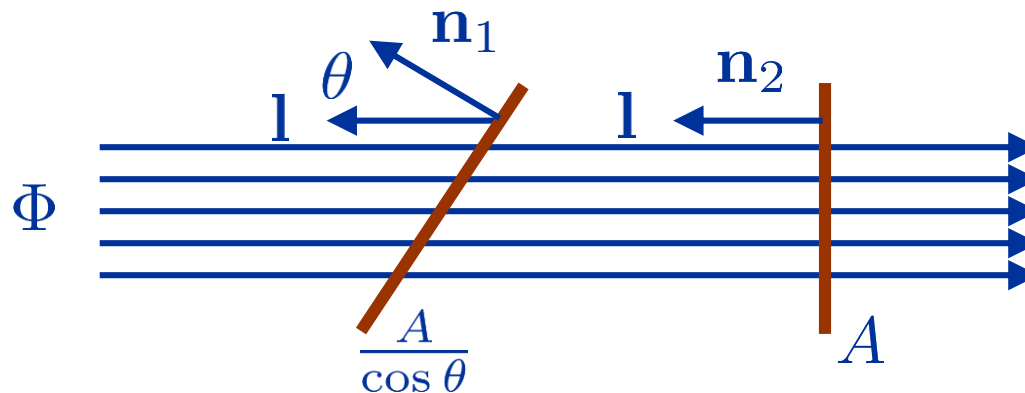
- irradiance on an oriented surface patch above a Lambertian surface is constant



$$E = \frac{\Phi}{dA} = \frac{\Phi \cos \theta}{dA \cos \theta}$$

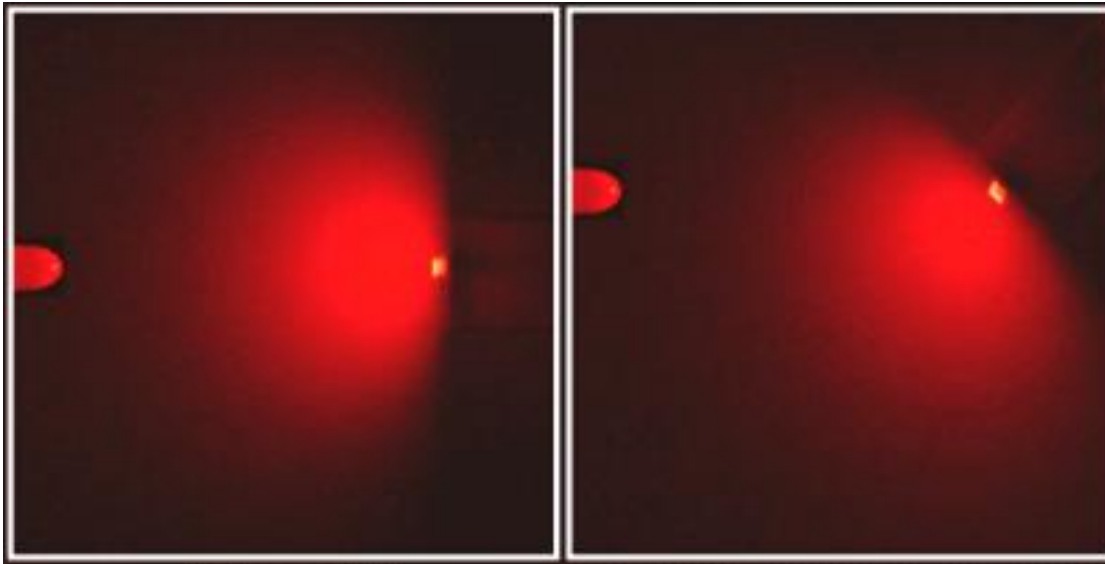
Lambert's Cosine Law

- irradiance on a surface is proportional to the cosine of the angle between surface normal \mathbf{n} and light source direction \mathbf{l} (also referred to as Lambert's Cosine Law)

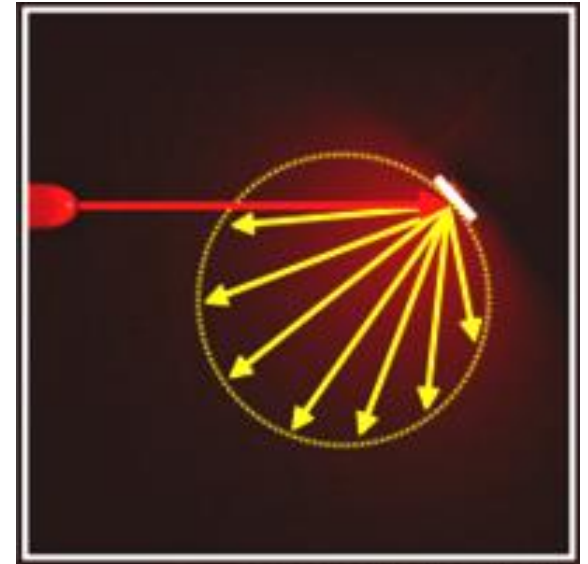


$$E = \frac{\Phi}{A / \cos \theta} = \frac{\Phi}{A} \cos \theta = E_L \cos \theta$$

Diffuse Lighting



Irradiance of the object surface (yellow) depends on the angle between light source (red) direction and surface normal.



Radiant intensity of the surface depends on the angle between radiation direction and surface normal.

[Wikipedia: Lambertsches Gesetz]

Diffuse Lighting

- radiance (RGB) is computed as $\mathbf{L}_{\text{diff}} = \frac{1}{\pi} \cdot \mathbf{c}_{\text{diff}} \otimes \mathbf{E}_L \cdot \cos \theta$
 - \otimes denotes component-wise multiplication,
 $(a, b, c) \otimes (d, e, f) = (a \cdot d, b \cdot e, c \cdot f)$
 - $1/\pi$ is a normalization coeff. motivated by energy conservation: light is reflected in all directions of a hemisphere and the viewer only receives a portion of the reflected light
- in implementations,
 - π is usually incorporated in \mathbf{E}_L
 - $\cos \theta$ is computed as the dot product of the **normalized** light direction and the **normalized** surface normal
 - $\mathbf{L}_{\text{diff}} = \mathbf{c}_{\text{diff}} \otimes \mathbf{E}_L \cdot \max(0, \mathbf{n} \cdot \mathbf{l})$
 - neg. values $\mathbf{n} \cdot \mathbf{l}$ correspond to illuminating the back of a surface, the max-function is often omitted for readability

Outline

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 - diffuse reflection
 - specular reflection
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 - miscellaneous
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Specular Reflection

- perceived radiance depends on the viewing direction
 - maximum radiance, if the viewer direction \mathbf{v} corresponds to the reflection direction \mathbf{r} or, if the halfway vector \mathbf{h} corresponds to the surface normal

- Phong model

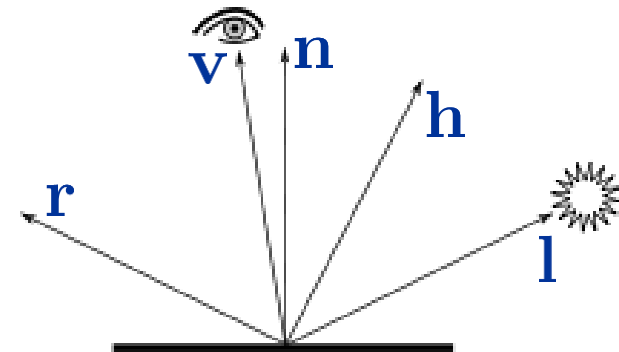
$$\mathbf{L}_{\text{spec}} = \mathbf{c}_{\text{spec}} \otimes \mathbf{E}_L \cdot (\max(0, \mathbf{v} \cdot \mathbf{r}))^m$$

$\mathbf{c}_{\text{spec}} = (1,1,1)$ is popular for the specular color, since the color of specular reflection converges to the color of the incident light. The exponent m characterizes the size of the specular highlight. Maximum radiance is not influenced.

- Blinn-Phong model

$$\mathbf{L}_{\text{spec}} = \mathbf{c}_{\text{spec}} \otimes \mathbf{E}_L \cdot (\max(0, \mathbf{n} \cdot \mathbf{h}))^m$$

Standard specular term in OpenGL (prior to OpenGL 3.1)

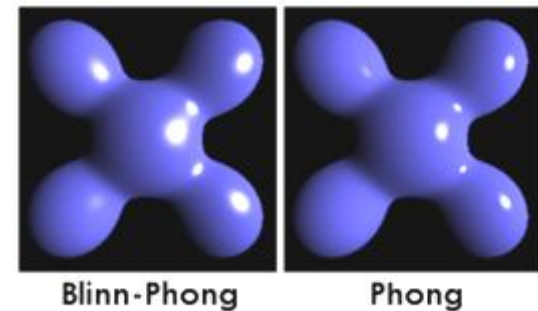


All these vectors are normalized in computations.

[Wikipedia:
Blinn-Phong shading model]

Specular Reflection Implementation

- Phong model
 - requires the reflection vector \mathbf{r} which can be computed from \mathbf{n} and \mathbf{l}
$$\mathbf{r} = 2(\mathbf{n} \cdot \mathbf{l})\mathbf{n} - \mathbf{l}$$
- Blinn-Phong model
 - requires the halfway vector \mathbf{h} which can be computed from \mathbf{l} and \mathbf{v}
$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{\|\mathbf{l} + \mathbf{v}\|}$$



[Wikipedia:
Blinn-Phong shading model]

Reflection Vector

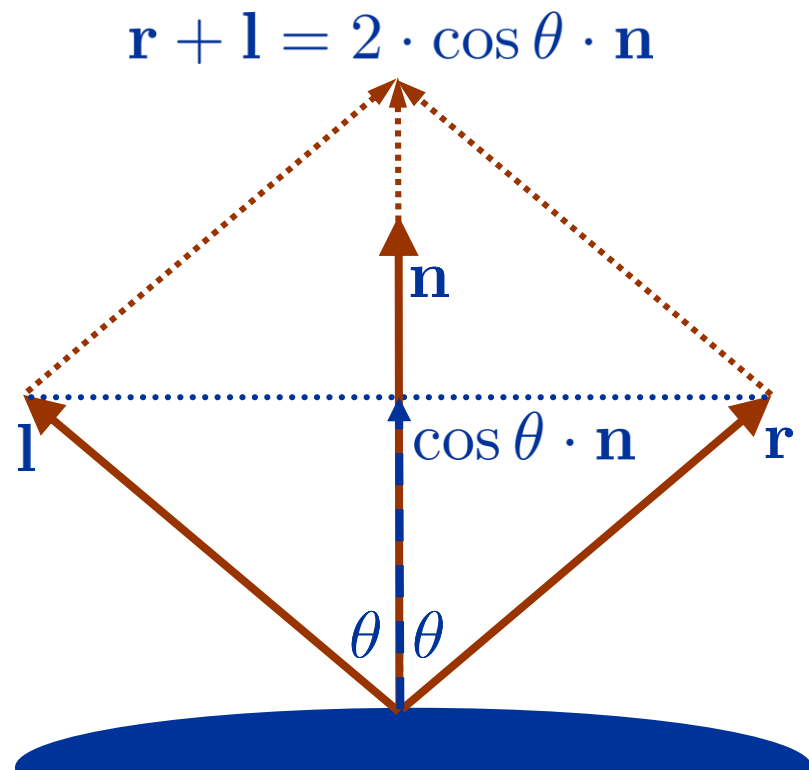
- computed with light source direction \mathbf{l} and surface normal \mathbf{n}

$$\mathbf{r} + \mathbf{l} = 2 \cdot \cos \theta \cdot \mathbf{n}$$

$$\cos \theta = \mathbf{l} \cdot \mathbf{n}$$

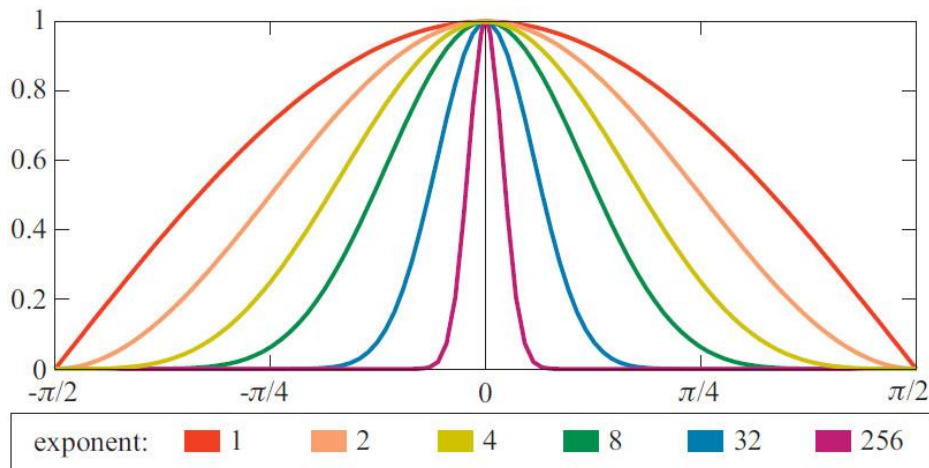
$$\mathbf{r} = 2 \cdot (\mathbf{l} \cdot \mathbf{n}) \cdot \mathbf{n} - \mathbf{l}$$

- \mathbf{l} and \mathbf{n} have to be normalized
- \mathbf{r} is normalized



Non-normalized Blinn-Phong

- Phong and Blinn-Phong do not account for energy preservation
- radiance depends on angle θ and exponent m
- radiant exitance from the surface depends on m



angle θ between v and r (Phong)
or n and h (Blinn-Phong)

[Akenine-Möller et al.]

Normalized Blinn-Phong

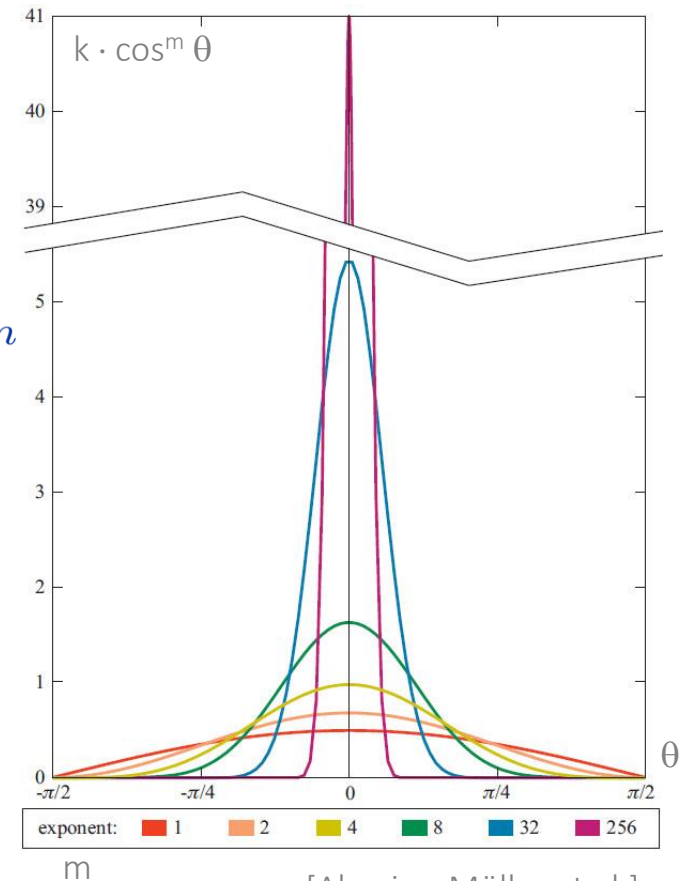
- normalized Blinn-Phong versions account for energy conservation

- e.g.

$$\mathbf{L}_{\text{spec}} = \frac{m+8}{8\pi} \cdot \mathbf{c}_{\text{spec}} \otimes \mathbf{E}_L \cdot (\mathbf{n} \cdot \mathbf{l}) \cdot (\mathbf{n} \cdot \mathbf{h})^m$$

[Akenine-Möller et al., Real-time Rendering]

- considers angle-dependent irradiance
- considers growing maximum radiance for growing exponent m
- radiant exitance is approximately constant for varying m



[Akenine-Möller et al.]

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Ambient Light

- diffuse and specular reflection are modeled for directional light from a point light source or from parallel light (point light source at infinity)
- ambient light is an approx. for indirect light sources
 - reflected light from objects illuminates other objects
 - all objects in the hemisphere seen from a surface act as light source for this surface
- this effect is generally approximated by a constant, object-dependant offset $\mathbf{L}_{\text{amb}} = \mathbf{c}_{\text{amb}} \otimes \mathbf{E}_{\text{amb}}$
- \mathbf{c}_{amb} usually corresponds to \mathbf{c}_{diff} , \mathbf{E}_{amb} represents ambient illumination, e.g. the dominant object color

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Phong Illumination Model

- combines ambient, diffuse, and specular components

$$\mathbf{L}_p = \mathbf{c}_{\text{amb}} \otimes \mathbf{E}_{\text{amb}} + \mathbf{c}_{\text{diff}} \otimes \mathbf{E}_{\text{diff}} \cdot (\mathbf{n} \cdot \mathbf{l}) + \mathbf{c}_{\text{spec}} \otimes \mathbf{E}_{\text{spec}} \cdot (\mathbf{r} \cdot \mathbf{v})^m$$

- Phong allows to set different light colors for different components (which is not physically motivated)

- a useful parameter setting could be

$$\mathbf{L}_p = \underbrace{\mathbf{E}_{\text{amb}}}_{\text{ambient illumination}} \otimes \underbrace{\mathbf{c}_{\text{obj}}}_{\text{object color}} + \underbrace{\mathbf{E}_{\text{light}}}_{\text{light color}} \otimes (\underbrace{\mathbf{c}_{\text{obj}}}_{\text{object color}} \cdot (\mathbf{n} \cdot \mathbf{l}) + \underbrace{\mathbf{c}_{\text{spec}}}_{\text{highlight color (e. g. white)}} \cdot (\mathbf{r} \cdot \mathbf{v})^m)$$

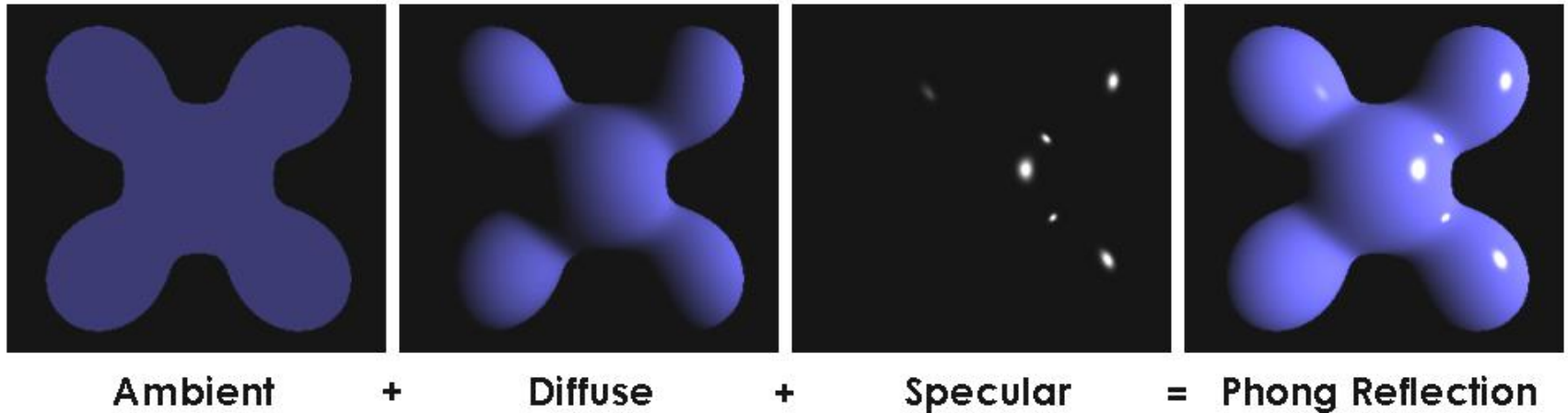
- multiple light sources, e. g.

$$\mathbf{L}_p = \mathbf{c}_{\text{amb}} \otimes \mathbf{E}_{\text{amb}} + \sum_{\text{light}} \mathbf{E}_{\text{light}} \otimes (\mathbf{c}_{\text{diff}} \cdot (\mathbf{n} \cdot \mathbf{l}) + \mathbf{c}_{\text{spec}} \cdot (\mathbf{r} \cdot \mathbf{v})^m)$$

- max-functions are omitted. \mathbf{n} , \mathbf{l} , \mathbf{r} , \mathbf{v} are normalized.

Max values for RGB comp. of \mathbf{L}_p have to be considered.

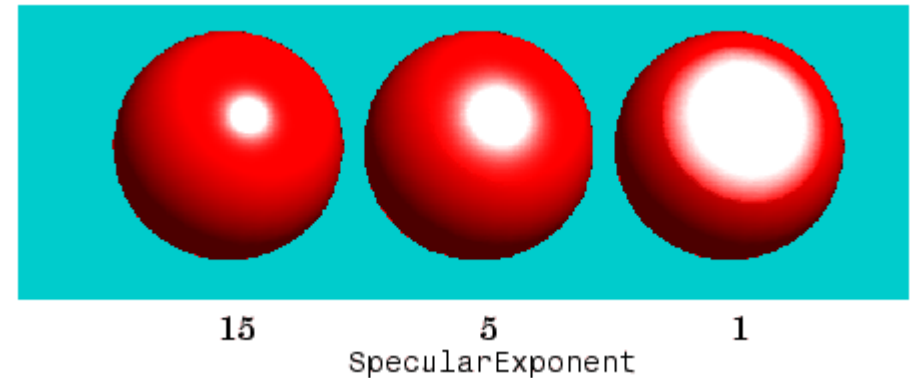
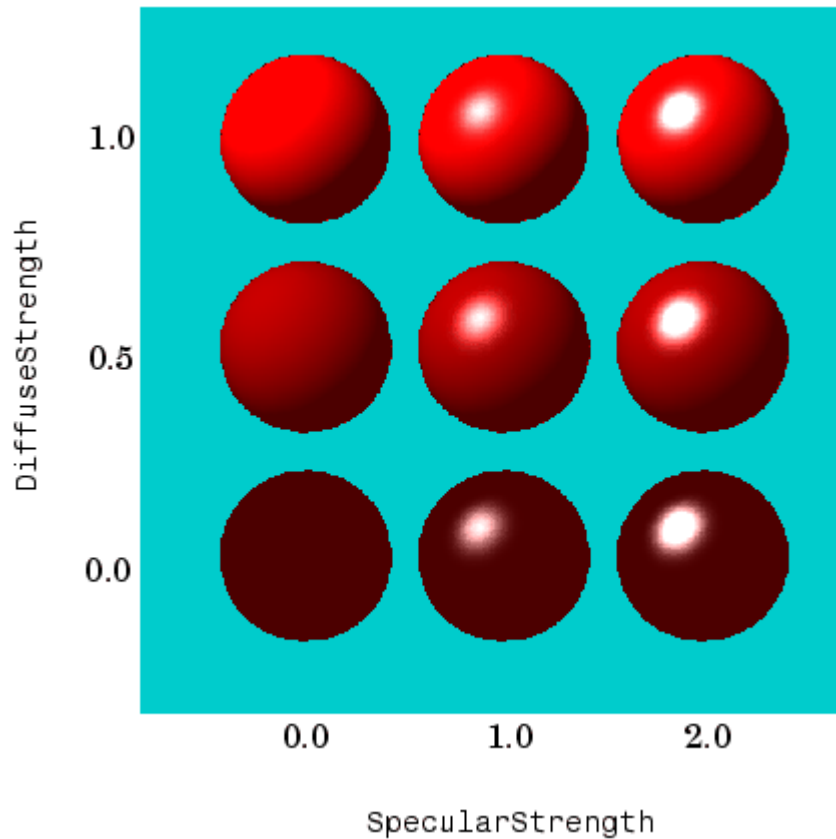
Phong Illumination Model



- parameters (material and light colors) can be used to adapt the ratios of ambient, diffuse, and specular reflection components

[Wikipedia: Phong shading]

Phong Illumination Model



[<http://www.mathworks.com/help/techdoc/visualize/f1-21818.html>]

Blinn-Phong Illumination Model

- non-normalized Blinn-Phong

$$\mathbf{L}_{bp} = \mathbf{c}_{amb} \otimes \mathbf{E}_{amb} + \mathbf{c}_{diff} \otimes \mathbf{E}_{diff} \cdot (\mathbf{n} \cdot \mathbf{l}) + \mathbf{c}_{spec} \otimes \mathbf{E}_{spec} \cdot (\mathbf{h} \cdot \mathbf{n})^m$$

- parameter setting and multiple light sources (see Phong)

- normalized Blinn-Phong, e.g.

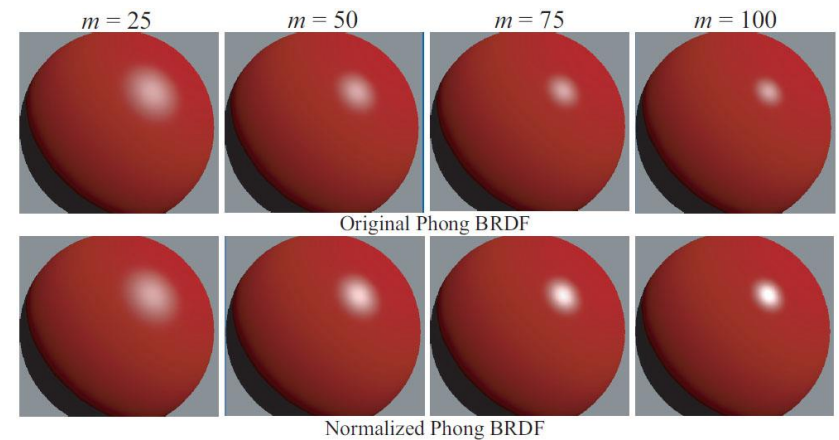
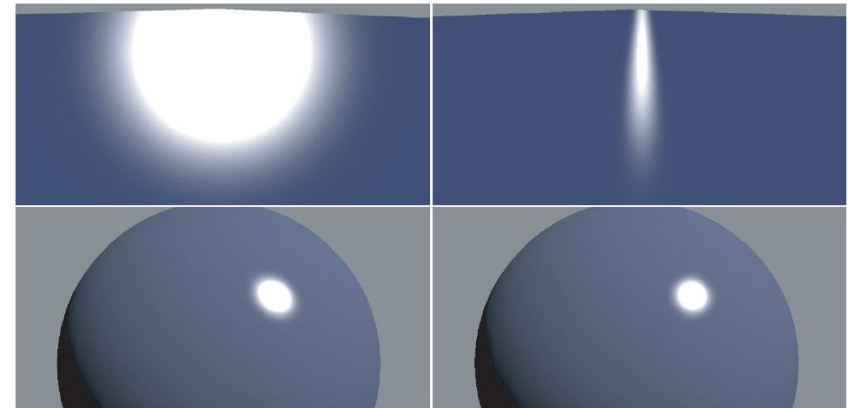
$$\mathbf{L}_{nbp} = \mathbf{E}_{amb} \otimes \mathbf{c}_{amb} + \left(\frac{1}{\pi} \mathbf{c}_{diff} + \frac{m+8}{8\pi} \cdot \mathbf{c}_{spec} \cdot (\mathbf{h} \cdot \mathbf{n})^m \right) \otimes \mathbf{E}_L \cdot (\mathbf{l} \cdot \mathbf{n})$$

- in implementations, the normalization coefficient can be incorporated into \mathbf{c}_{diff} and \mathbf{c}_{spec}

$$\mathbf{L}_{nbp} = \mathbf{E}_{amb} \otimes \mathbf{c}_{amb} + (\mathbf{c}_{diff} + \mathbf{c}_{spec} \cdot (\mathbf{h} \cdot \mathbf{n})^m) \otimes \mathbf{E}_L \cdot (\mathbf{l} \cdot \mathbf{n})$$

Phong vs. Blinn-Phong

- Phong (left) and Blinn-Phong (right)
- highlights on flat surfaces are more realistic with Blinn-Phong
- Blinn-Phong (top) and normalized Blinn-Phong (bottom)
- maximum radiance of the highlight for varying m is constant for Blinn-Phong



[Akenine-Möller et al.]

Outline

- light
- color
- lighting models
 - diffuse reflection
 - specular reflection
 - ambient light
 - Phong illumination model
 - miscellaneous
- shading models

Considering Distances

- between object surface and light source
 - irradiance of a surface illuminated by a point light source is inversely proportional to the squared distance from the surface to the light source
 - light source attenuation
- between object surface and viewer
 - atmospheric effects, e. g. fog, influence the visibility of objects
 - visibility refers to the transparency of air
 - if air is transparent, objects are clearly visible
 - in less transparent air, fog particles absorb some flux and scatter additional flux towards the viewer
 - in low visibility, radiance converges to a "fog color"

Light Source Attenuation

- for a point light source with position $\mathbf{l} = (l_x, l_y, l_z, 1)^T$, the distance d from the light source to a surface point $\mathbf{p} = (p_x, p_y, p_z, 1)^T$ can be considered in the irradiance of the light source

$$d = \|\mathbf{p} - \mathbf{l}\|$$

$$\mathbf{E}_{\text{att}} = \frac{1}{k_c + k_l \cdot d + k_q \cdot d^2} \mathbf{I}_{\text{light}}$$

- this is motivated by $\mathbf{E}_{\text{light}} = \frac{\mathbf{I}_{\text{light}}}{d^2}$
(irradiance \mathbf{E} is inversely proportional to the squared distance from the light source with radiant intensity \mathbf{I})
- k_c, k_l, k_q are user-defined parameters

Fog

- fog is approximated by a linear combination of the computed radiance \mathbf{L} and a fog color \mathbf{c}_{fog}
- d is the distance of the surface point to the viewer (its z-component, i.e. depth value)
$$\mathbf{L}_{\text{fog}} = f(d) \cdot \mathbf{L} + (1 - f(d)) \cdot \mathbf{c}_{\text{fog}}$$
- $0 \leq f(d) \leq 1$ is a function that describes the visibility
 - $f(d) = 1$: max visibility (object color is unaffected)
 - $f(d) = 0$: min visibility (object color is changed to fog color)
 - e. g. $f(d) = \frac{d_{\text{end}} - d}{d_{\text{end}} - d_{\text{start}}}$ $f(d) = e^{-\text{density} \cdot d}$

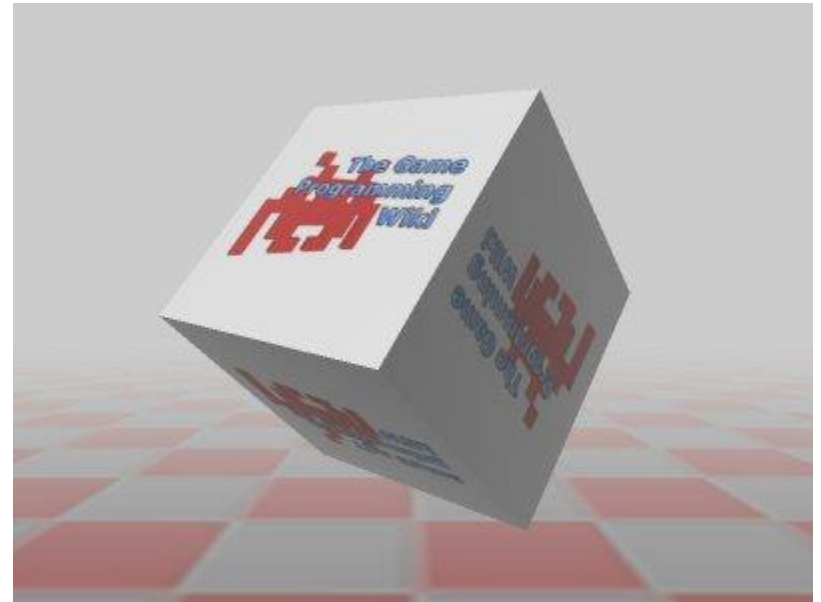
linear fog: starts at d_{start} ,
minimum visibility at d_{end} .
Clamped to $[0..1]$.

exponential fog

Attenuation and Fog



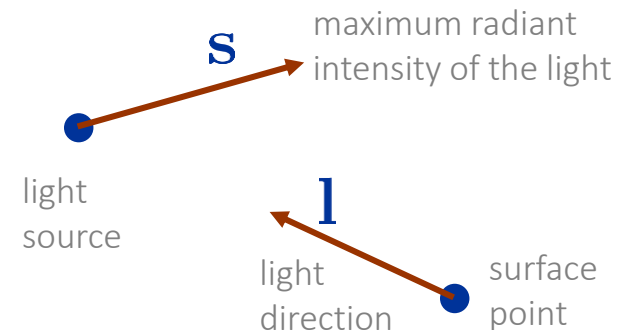
[<http://www.gamedev.net/topic/541383-typical-light-attenuation-coefficients/>]



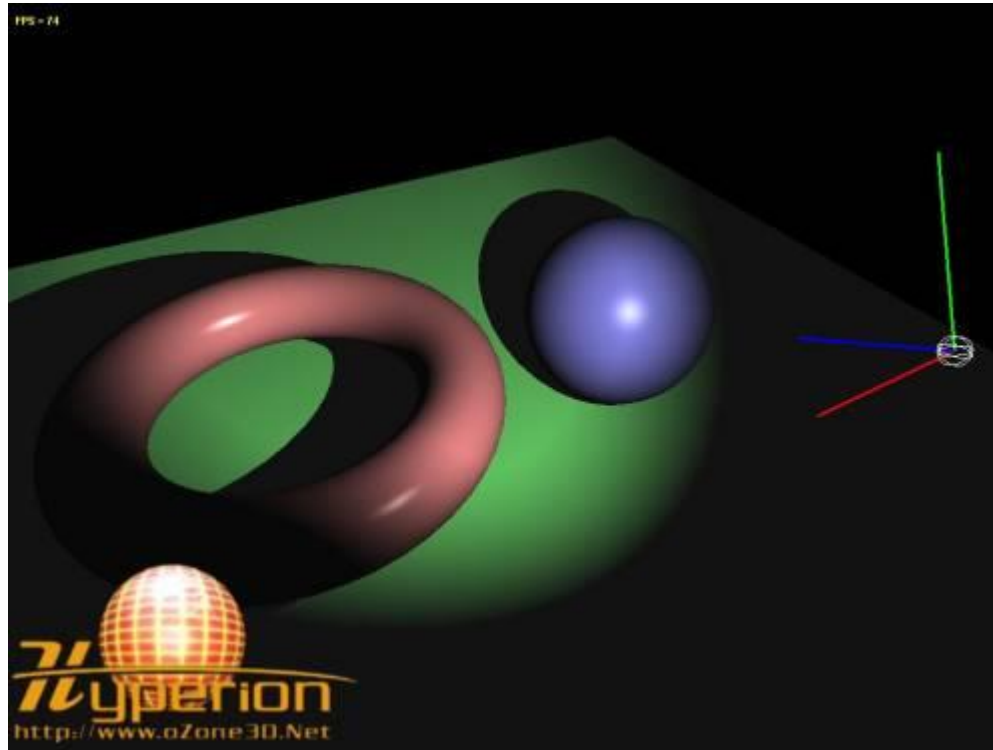
[The Game Programming Wiki:
OpenGL:Tutorials:Tutorial
Framework:Light and Fog]

Light Sources

- positional light source at position $\mathbf{l} = (l_x, l_y, l_z, 1)^T$
- directional light source with direction $\mathbf{l} = (l_x, l_y, l_z, 0)^T$
- spotlight
 - positional light with flux into restricted directions
 - e. g. $\mathbf{E}_{\text{spot}} = \mathbf{I}_{\text{light}} \cdot \max(-\mathbf{l} \cdot \mathbf{s}, 0)^{m_{\text{spot}}}$
 - $\mathbf{I}_{\text{light}}$ is the maximum radiant intensity of the spotlight in direction \mathbf{s}
 - \mathbf{l} is the direction to the light
 - m_{spot} is user-defined to adapt the fall-off rate of the radiant intensity with respect to the cosine of the angle between $-\mathbf{l}$ and \mathbf{s}
 - ($\mathbf{I}_{\text{light}}$ should be divided by some squared distance)



Spotlight



[http://www.ozone3d.net/tutorials/glsl_lighting_phong_p3.php]

Summary

- the class of Phong lighting models considers
 - diffuse and specular reflection
 - ambient illumination
- the lighting models are efficient to compute as they only consider local information, e.g. surface normal, light source direction, viewer direction, ...
- improved variants consider energy conservation and lead to more realistic specular highlights
- additionally, distances to the viewer and the light source can be considered
- various types of light sources can be realized

Outline

- light
- color
- lighting models
- shading models

Introduction

- shading models specify whether the lighting model is evaluated **per vertex or per fragment**
- if evaluated per vertex, the shading model specifies whether the resulting vertex **colors are interpolated** across a primitive **or not**
- if evaluated per fragment, surface **normals are interpolated** across a primitive

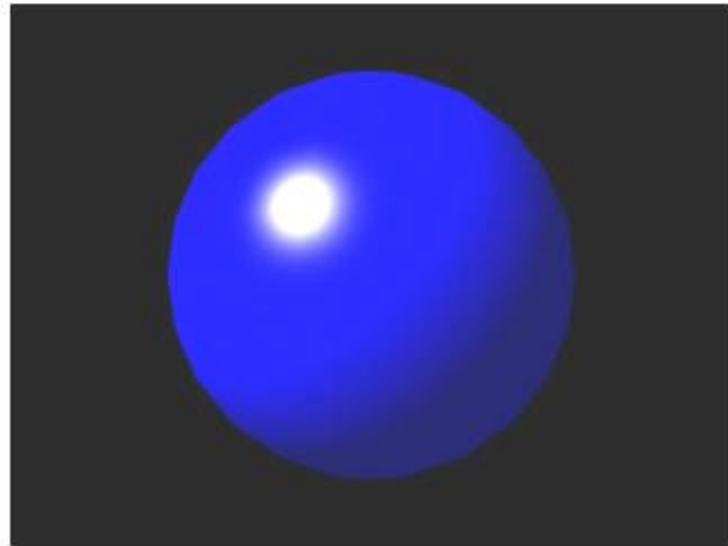
Shading Models

- flat shading (constant shading)
 - evaluation of the lighting model per vertex
 - primitives are colored with the color of one specific vertex
- Gouraud shading
 - evaluation of the lighting model per vertex
 - primitives are colored by bilinear interpolation of vertex colors
- Phong shading
 - bilinear interpolation of vertex normals during rasterization
 - evaluation of the lighting model per fragment

Flat vs. Phong



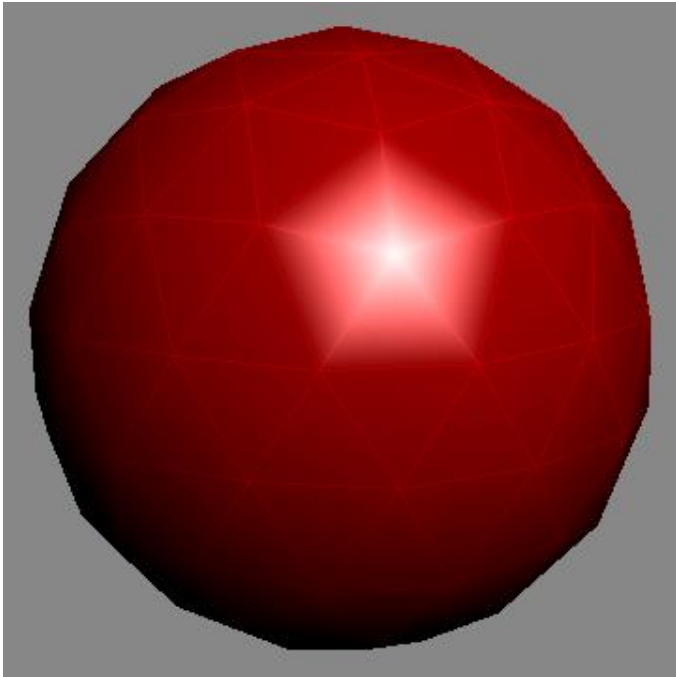
FLAT SHADING



PHONG SHADING

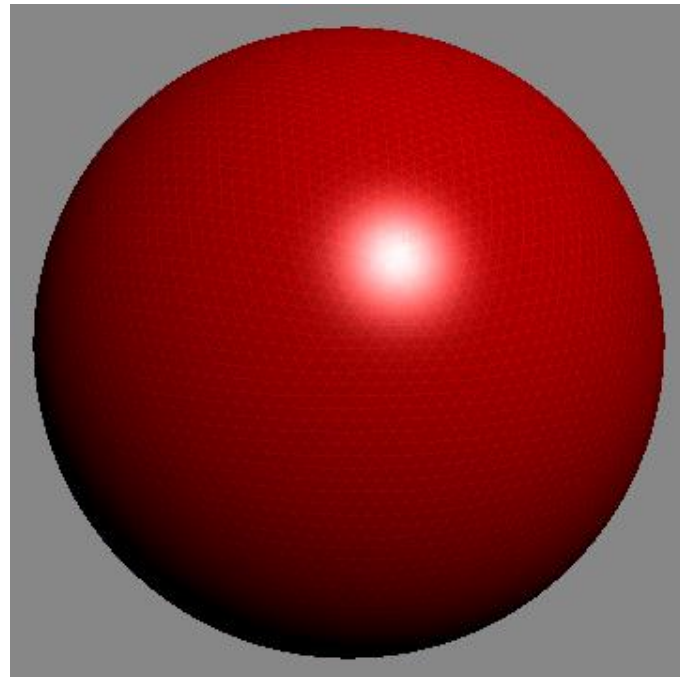
[Wikipedia: Phong shading]

Gouraud Shading



low polygon count

Highlight is poorly resolved.
Mach band effect.

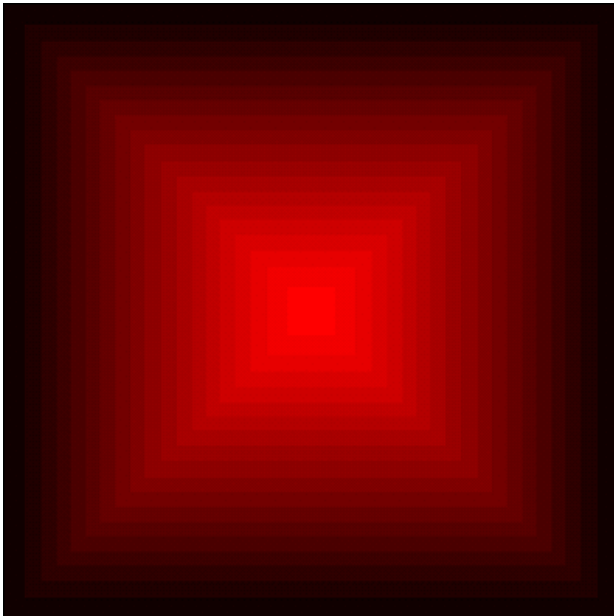


high polygon count

[Wikipedia: Gouraud shading]

Mach Band Effect

- mach bands are illusions due to our neural processing



The bright bands at 45 degrees and 135 degrees are illusory.

The intensity inside each square is the same.

Shading Models

- flat shading (constant shading)
 - simplest, fastest
- Gouraud shading
 - more realistic than flat shading for the same tessellation
 - suffers from Mach band effect
 - local highlights are not resolved, if the highlight is not captured by a vertex
- Phong shading
 - highest quality, most expensive

Summary

- light
- color
- lighting models
- shading models