Computer Graphics
Rasterization

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Outline

- Context
- Rasterization-based rendering
- Vertex processing
- Rasterization
- Fragment processing
- Framebuffer update
What is visible at the sensor?

- Visibility can be resolved by ray casting or by rasterization

Ray casting computes ray-scene intersections to estimate \( q \) from \( p_1 \) and \( p_2 \).

Rasterizers apply transformations to \( p_1 \) and \( p_2 \) in order to estimate \( q \).

- If more than one scene point \( p_i \) is mapped to the same sensor position \( q \), the scene point closest to the viewer is selected.
**Rasterization**

- Computation of pixel positions in an image plane that represent a projected primitive

Primitives represented by vertices

- Triangle (3 vertices)
- Line segment (2 vertices)
- Vertex

Image plane / 2D array of pixels
Rasterization and Visibility

- After rasterization, visibility can be efficiently resolved per pixel position
  - Distances of primitives to the viewer, i.e. depth values, can be compared per pixel position
Rasterization and Canonical View Volume

- Rasterization is typically implemented for canonical view volumes.

Side view of a scene in a canonical view volume

Rasterization result

Resolved visibility
Rasterization and Rendering

- Rasterization is typically embedded in a complete rendering approach
  - Rendering pipeline
  - Rasterization-based rendering
  - Rasterization
Rasterization-based Rendering

\[ PV^{-1}M_i \]

Object in local space  
Scene in the canonical view volume  
Rasterization  
Visibility  
Shading  
Image
Vertices: have positions and other attributes.
Primitives: are represented by vertices.
Fragments: are pixel candidates with pixel positions and other attributes.
Pixels: have a position and other attributes, in particular color.
Framebuffer: consists of pixels.
Terms - Illustration

Triangle 1 with three vertices

Rasterizer generates fragments.

Frames of the framebuffer

Fragment attributes are used to update pixel attributes in the framebuffer.

Triangle 2 with three vertices

Rasterizer generates fragments.

Frames of the framebuffer

Framebuffer attributes can be updated. Fragments can be discarded.
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Main Stages

- **Vertex processing**
  - Input: Vertices
  - Output: Vertices
  - Transformations
  - Setting, computation, processing of vertex attributes, e.g. position, color (Phong), texture coordinates
Main Stages

- **Rasterization**
  - Input: Vertices and connectivity information
  - Output: Fragments
  - Primitive assembly
  - Rasterization of primitives
    - Generates fragments from vertices and connectivity information
  - Sets or interpolates fragment attributes from vertex attributes, e.g. distance to viewer (depth), color, texture coordinates
Main Stages

- Fragment processing
  - Input: Fragments
  - Output: Fragments
  - Fragment attributes are processed, e.g. color
  - Fragments can be discarded

- Framebuffer update
  - Input: Fragments
  - Output: Framebuffer attributes
  - Fragment attributes update framebuffer attributes, e.g. color
Main Stages - Overview

Framebuffer Update

[Image of diagram showing stages of rendering]
Discussion

- Realization motivated by computational efficiency
  - Vertices and fragments are processed independently in the respective stages
- Stages are supported by graphics hardware GPU
  - OpenGL, DirectX, Vulkan are software interfaces to GPUs
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Transformations of Vertex Positions

- Scene modeling
  - Object placement $M_i$
  - Camera placement $V$
  - Internal camera parameters, i.e. focal length $P$
- Vertices $p$ of object $i$ are transformed with
  $$p' = PV^{-1}M_i p$$
Transformations of Vertex Positions

- GPU rasterizers assume that all vertex positions are in clip / NDC space.
- Only vertices inside the canonical view volume, e.g. \((-1 \ldots 1, -1 \ldots 1, -1 \ldots 1)\), are processed.
- Transformation \(p' = PV^{-1}M_ip\) can realize user-defined scene setups.
- Alternatively, the scene can be setup within the canonical view volume and rendered with parallel projection. Then, transformations are not required.
Vertex Attributes

- Position \((p_x, p_y, p_z, 1)^T\)
  - Z-component in NDC space is referred to as depth value. Represents distance to the camera plane.
- Color \((R, G, B, A)^T\)
  - Can optionally be defined or computed with Phong, if surface normal, light and material properties are available
  - \(A\) can be used for rendering effects, e.g. transparency
- Texture coordinates, e.g. \((u, v)\)
  - For lookup and processing of additional data, i.e. textures
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Rasterization

- **Input**
  - Vertices with connectivity information and attributes
    - Color, depth, texture coordinates

- **Output**
  - Fragments with attributes
    - Pixel position
    - Interpolated color, depth, texture coordinates
Line Rasterization

- Line begins and ends at integer-valued positions \( p_b = (x_b, y_b) \) and \( p_e = (x_e, y_e) \)
- Algorithm defined for line slopes between 0 and 1
  - Generalized by employing symmetries
- One fragment per integer \( x \)-value
  - First fragment: \( (x_b, y_b) \)
  - Next fragment: \( (x_b + 1, y_b) \) or \( (x_b + 1, y_b + 1) \)
  - Last fragment: \( (x_e, y_e) \)

[Wikipedia: Rasterung von Linien]
Bresenham Line Algorithm

- Based on the current fragment \((x_i, y_i)\), the algorithm decides whether to choose \((x_i + 1, y_i)\) or \((x_i + 1, y_i + 1)\).
- Line representation: \(F(x, y) = ax + by + c = 0\).
- \(F\) is evaluated at the midpoint between \((x_i + 1, y_i)\) and \((x_i + 1, y_i + 1)\).
- \(F(x_i + 1, y_i + \frac{1}{2}) > 0\)
  choose \(NE\), i.e. \((x_i + 1, y_i + 1)\).
- \(F(x_i + 1, y_i + \frac{1}{2}) \leq 0\)
  choose \(E\), i.e. \((x_i + 1, y_i)\).

[Wikipedia: Rasterung von Linien]
Incremental Update of the Decision Variable

- Decision variable \( d_i = F(x_i + 1, y_i + \frac{1}{2}) \)
- Incremental update from \( d_i \) to \( d_{i+1} \)
  - \( d_i > 0 \) ⇒ choose \( NE \), \( d_{i+1} = F(x_i + 2, y_i + 1 + \frac{1}{2}) \)
  - \( d_i \leq 0 \) ⇒ choose \( E \), \( d_{i+1} = F(x_i + 2, y_i + \frac{1}{2}) \)
- In case of \( d_i > 0 \):
  \[
  \Delta_{NE} = d_{i+1} - d_i = \Delta y \cdot (x_i + 2) - \Delta x \cdot (y_i + \frac{3}{2}) + c - (\Delta y \cdot (x_i + 1) - \Delta x \cdot (y_i + \frac{1}{2}) + c)
  \]
  \[
  \Delta_{NE} = \Delta y - \Delta x
  \]
- In case of \( d_i \leq 0 \):
  \[
  \Delta_E = d_{i+1} - d_i = \Delta y \cdot (x_i + 2) - \Delta x \cdot (y_i + \frac{1}{2}) + c - (\Delta y \cdot (x_i + 1) - \Delta x \cdot (y_i + \frac{1}{2}) + c)
  \]
  \[
  \Delta_E = \Delta y
  \]

\[
F(x, y) = ax + by + c
\]
\[
a = \Delta y = y_e - y_t
\]
\[
b = -\Delta x = x_b - x_e
\]
Bresenham Algorithm - Initialization

− For start fragment $p_b = (x_b, y_b)$, the decision variable can be initialized as

\[ d_1 = F(x_b + 1, y_b + \frac{1}{2}) = \Delta y \cdot (x_b + 1) - \Delta x \cdot (y_b + \frac{1}{2}) + c \]

\[ = \Delta y \cdot x_b - \Delta x \cdot y_b + c + \Delta y - \frac{1}{2} \Delta x \]

\[ = F(x_b, y_b) + \Delta y - \frac{1}{2} \Delta x \]

\[ = \Delta y - \frac{1}{2} \Delta x \]

− Floating-point arithmetic is avoided by considering $2 \cdot F(x, y)$:

\[ d_1 = 2\Delta y - \Delta x \]

\[ \Delta_E = 2\Delta y \]

\[ \Delta_{NE} = 2\Delta y - 2\Delta x \]
Bresenham Algorithm - Implementation

```c
void BresenhamLine(int xb, int yb, int xe, int ye) {

    int dx, dy, incE, incNE, d, x, y;

    dx = xe - xb; dy = ye - yb;
    d = 2*dy - dx; incE = 2*dy; incNE = 2*(dy - dx);
    x = xb; y = yb;

    GenerateFragment(x, y);

    while (x < xe) {
        x++;
        if (d <= 0) d += incE; /* choose E */
        else {d += incNE; y++;} /* choose NE */
        GenerateFragment(x, y);
    }
}
```
Polygon Rasterization

- Compute intersections of non-horizontal polygon edges with horizontal scanlines
- Intersections are computed for scanlines $y = y_i + 0.5$
- Fill pixel positions in-between two intersections with fragments
  - Scan from left to right
  - Enter the polygon at the first intersection, leave the polygon at the next intersection
- Works for closed polygons

[Wikipedia: Rasterung von Polygonen]
Polygon Rasterization

- For each polygon edge
  - Process all scanlines intersected by the edge
  - Invert all positions with an x-component larger than the intersection point

[Wikipedia: Rasterung von Polygonen]
Attribute Interpolation

– Attributes are interpolated from vertices to fragments

– Challenge in case of perspective projection: Linear interpolation in view space cannot be realized by linear interpolation in clip space
Attribute Interpolation

Perspective projection of a line $AB$. $t / (1-t)$ is not equal to $s / (1-s)$. Therefore, linear interpolation in clip space between $a$ and $b$ does not correspond to a linear interpolation between $A$ and $B$ in view space.

$$I_t = I_1 + t(I_2 - I_1)$$
Linear interpolation in view space

$$I_s = I_1 + \frac{sZ_1}{sZ_1 + (1-s)Z_2}(I_2 - I_1)$$
Non-linear interpolation in clip space

$$I_s = \frac{I_1}{Z_1} + s\left(\frac{I_2}{Z_2} - \frac{I_1}{Z_1}\right)$$
Linear interpolation of $I/Z$ and $1/Z$ in clip space

[Kok-Lim Low: Perspective-Correct Interpolation]
Attribute Interpolation

- Perspective projection transform

\[
\begin{pmatrix}
    x_{\text{clip}} \\
    y_{\text{clip}} \\
    z_{\text{clip}} \\
    w_{\text{clip}}
\end{pmatrix}
= \begin{pmatrix}
    \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\
    0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\
    0 & 0 & \frac{f+n}{f-n} & -2fn/(f-n) \\
    0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
    X_{\text{view}} \\
    Y_{\text{view}} \\
    Z_{\text{view}} \\
    1
\end{pmatrix}
\]

- Linear relation between \( w_{\text{clip}} \) in clip space and \( z_{\text{view}} \) in view space \( w_{\text{clip}} = Z_{\text{view}} \)

- \( Z_{\text{view}} \) or \( w_{\text{clip}} \) can be used in the interpolation

In view space: \[
I_s = \frac{\frac{1}{Z_1} + s\left(\frac{I_2}{Z_2} - \frac{I_1}{Z_1}\right)}{\frac{1}{Z_1} + s\left(\frac{1}{Z_2} - \frac{1}{Z_1}\right)}
\]

In clip space: \[
I_s = \frac{\frac{1}{w_1} + s\left(\frac{I_2}{w_2} - \frac{I_1}{w_1}\right)}{\frac{1}{w_1} + s\left(\frac{1}{w_2} - \frac{1}{w_1}\right)}
\]
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Fragment Processing

Vertices with connectivity and attributes

Fragments (pixel candidates) with attributes

Pixels of the framebuffer with attributes, in particular color

Fragment attributes are processed
**Fragment Processing**

- Fragment attributes are processed
- Fragment attributes are tested
  - Fragments can be discarded
  - Fragments can pass a test and fragment attributes can be used to update framebuffer attributes
- Processing and testing make use of
  - Fragment attributes (position, color, depth, texture coord)
  - Textures (n dimensional arrays of data)
  - Framebuffer data that is available for each pixel position
    - Depth buffer, color buffer, stencil buffer, accumulation buffer
Fragment Processing

- Vertices of a primitive
- Connectivity
- Rasterization
- Texture - Color - Depth...
- Additional data
- Fragment - Color - Depth...
- Framebuffer - Color - Depth...
- Final image
- Pixel candidates

How vertices are connected to a primitive
Attribute Processing - Examples

- Texturing
  - Combination of fragment color and texture data
- Fog
  - Adaptation of fragment color using fog color and fragment depth
- Antialiasing
  - Adaptation of fragment alpha value
Texturing

Texture

Textured object
Texturing – 2D Example

Texture coordinates are typically defined or computed for vertices. Rasterizer interpolates texture coordinates from vertices to fragments.

$\begin{pmatrix} u_2, v_2 \\ u_1, v_1 \end{pmatrix}$

$\begin{pmatrix} u_3, v_3 \end{pmatrix}$

$d(u_f, v_f)$ is used for processing the attributes of the fragment.

$\begin{pmatrix} u_f, v_f \end{pmatrix}$

Texture data $d$
Tests - Examples

- Scissor test
  - Check if fragment position is inside a specified rectangle
- Alpha test
  - Check range of the fragment alpha value
  - Used for, e.g., transparency and billboarding
- Stencil test
  - Check if framebuffer stencil value at the fragment position fulfills a certain requirement
  - Used for, e.g., shadows
Depth Test – Resolving Visibility

- Depth test
  - Compare fragment depth value with the framebuffer depth value at the fragment position
  - If the fragment depth value is larger than the framebuffer depth value, the fragment is discarded
  - If the fragment depth value is smaller than the framebuffer depth value, the fragment passes and its attributes replace the current color and depth values in the framebuffer
Depth Test

Current framebuffer

Incoming fragments triangle 1

Updated framebuffer

Current framebuffer

Incoming fragments triangle 2

Updated framebuffer
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Blending

− Combines the fragment color $C_{fr}$ with the framebuffer color $C_{fb}(x_{fr})$ at the fragment position $x_{fr}$

$$C_{fb}(x_{fr}) = \alpha_{fr} \cdot C_{fr} + (1 - \alpha_{fr}) \cdot C_{fb}(x_{fr})$$
Summary

- Rasterization combined with a depth test can resolve visibility
- Rendering pipeline employs rasterization
  - Vertex processing
  - Rasterization
  - Fragment processing
  - Framebuffer update
- Implemented on graphics hardware