Cloth Simulation

Simulation in Computer Graphics
University of Freiburg

WS 05/06

D. Baraff, A. Witkin. Large steps in cloth simulation. *Siggraph'98*, pp. 43-54, 1998

Acknowledgement

This slide set is based on the following sources:

- D. Baraff, A. Witkin. Large steps in cloth simulation. *Siggraph'98*, pp. 43-54, 1998.
- D. Macri. Real-time cloth. *Game developer conference*, 2000.
- D. Pritchard. Implementing Baraff and Witkin's cloth simulation. http://freecloth.enigmati.ca/, 2003.
- sourceforge. freecloth project. http://freecloth.enigmati.ca/
- R. Bridson. Computational aspects of dynamic surfaces. PhD thesis, Stanford University, 2003.

Outline

- geometry
- stretch forces
- shear forces
- bend forces
- dynamic simulation
- strain limiting
- alternative bend forces
- demonstrations
- discussion

Geometry

- triangulated mesh of mass points
- two representations of point positions
  - world coordinates $\mathbf{x} \in \mathbf{R}^3$
  - plane coordinates $(u, v) \in \mathbf{R}^2$
- world coordinates vary during the simulation
- plane coordinates represent initial, undeformed geometry
- $\mathbf{x} = \mathbf{w}(u, v)$ maps from plane to world coordinates

Stretching

- represented with partial derivatives $\mathbf{w}_u = \frac{\partial \mathbf{w}}{\partial u}$ $\mathbf{w}_v = \frac{\partial \mathbf{w}}{\partial v}$
- material is unstretched / uncompressed
  - in *u* direction iff $||\mathbf{w}_u|| = 1$
  - in *v* direction iff $||\mathbf{w}_v|| = 1$

- Example.

$$\mathbf{w}(u,v) = \begin{pmatrix} \frac{\sqrt{2}}{2}u \\ \frac{\sqrt{2}}{2}u \\ 0 \end{pmatrix} \rightarrow \frac{\partial \mathbf{w}(u,v)}{\partial u} = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{pmatrix} \quad ||\mathbf{w}_u|| = 1$$

$$\rightarrow ||\mathbf{w}(u_1,v) - \mathbf{w}(u_2,v)|| = ||u_1 - u_2||$$

Stretch of a Triangle

- mass points of a triangle $\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k$
- edges in world space $\Delta\mathbf{x}_1 = \mathbf{x}_j - \mathbf{x}_i$, $\Delta\mathbf{x}_2 = \mathbf{x}_k - \mathbf{x}_i$
- edges in the plane

$$\begin{pmatrix} \Delta u_1 \\ \Delta v_1 \end{pmatrix} = \begin{pmatrix} u_j - u_i \\ v_j - v_i \end{pmatrix} \qquad \begin{pmatrix} \Delta u_2 \\ \Delta v_2 \end{pmatrix} = \begin{pmatrix} u_k - u_i \\ v_k - v_i \end{pmatrix}$$

- **w** is approximated as linear function over a triangle
- $\rightarrow$ $\mathbf{w}_u$ and $\mathbf{w}_v$ are constant over a triangle

Stretch of a Triangle

- $\rightarrow$ $\Delta \mathbf{x}_1 = \mathbf{w}_u \Delta u_1 + \mathbf{w}_v \Delta v_1$ $\Delta \mathbf{x}_2 = \mathbf{w}_u \Delta u_2 + \mathbf{w}_v \Delta v_2$
- solving for $\mathbf{w}_u$ and $\mathbf{w}_v$

$$\begin{pmatrix} \mathbf{w}_u & \mathbf{w}_v \end{pmatrix} = \begin{pmatrix} \Delta \mathbf{x}_1 & \Delta \mathbf{x}_2 \end{pmatrix} \begin{pmatrix} \Delta u_1 & \Delta u_2 \\ \Delta v_1 & \Delta v_2 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} \Delta \mathbf{x}_1 & \Delta \mathbf{x}_2 \end{pmatrix} \frac{1}{\Delta u_1 \Delta v_2 - \Delta v_1 \Delta u_2} \begin{pmatrix} \Delta v_2 & -\Delta u_2 \\ -\Delta v_1 & -\Delta u_1 \end{pmatrix}$$

- we are not interested in **w**
  we just use $\mathbf{w}_u$ and $\mathbf{w}_v$ to represent *u* and *v* stretch
- does not work with degenerated triangles

Stretch Condition

$$\mathbf{C}_{st}(\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k) = \begin{pmatrix} C_{st,u} \\ C_{st,v} \end{pmatrix} = A \begin{pmatrix} \|\mathbf{w}_u(\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k)\| - 1 \\ \|\mathbf{w}_v(\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k)\| - 1 \end{pmatrix}$$

- measures stretch for a triangle in *u* and *v* direction
- equals zero iff the triangle is unstretched / uncompressed
- condition is weighted with the initial triangle area *A*

$$A = \frac{1}{2} \left\| \begin{pmatrix} \Delta u_1 \\ \Delta v_1 \\ 0 \end{pmatrix} \times \begin{pmatrix} \Delta u_2 \\ \Delta v_2 \\ 0 \end{pmatrix} \right\| = \frac{1}{2}(\Delta u_1 \Delta v_2 - \Delta v_1 \Delta u_2)$$

Stretch Energy and Forces

- stretch energy of a triangle

$$E_{st}(\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k) = \frac{1}{2} k_{st} \mathbf{C}_{st}^T \mathbf{C}_{st}$$

- user-defined k_{st}
- E_{st}>0 if the triangle is stretched or compressed
- stretch forces **F** at all mass points of a triangle

$$\mathbf{F}_{st}^{\{i,j,k\}}(\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k) = -\frac{\partial E_{st}}{\partial \mathbf{x}_{\{i,j,k\}}} = -k_{st} \frac{\partial \mathbf{C}_{st}}{\partial \mathbf{x}_{\{i,j,k\}}} \mathbf{C}_{st}$$

- forces are negative gradient of the energy in terms of mass point positions

Stretch Force

- stretch force at point $\mathbf{x}_i$

$$\mathbf{F}^i_{st} = -k_{st}\frac{\partial \mathbf{C}_{st}}{\partial \mathbf{x}_i}\mathbf{C}_{st} = -k_{st}\begin{pmatrix} \frac{\partial C_{st,u}}{\partial x_{i,x}} & \frac{\partial C_{st,v}}{\partial x_{i,x}} \\ \frac{\partial C_{st,u}}{\partial x_{i,y}} & \frac{\partial C_{st,v}}{\partial x_{i,y}} \\ \frac{\partial C_{st,u}}{\partial x_{i,z}} & \frac{\partial C_{st,v}}{\partial x_{i,z}} \end{pmatrix}\begin{pmatrix} C_{st,u} \\ C_{st,v} \end{pmatrix}$$

Stretch Force

- rewriting the derivatives of **C** in terms of **w**

$$\mathbf{F}_{st}^{i} = -k_{st} \left(\frac{A}{\left\| \mathbf{w}_u \right\|} \begin{pmatrix} \frac{\partial \mathbf{w}_u}{\partial \mathbf{x}_{i,x}} \mathbf{w}_u \\ \frac{\partial \mathbf{w}_u}{\partial \mathbf{x}_{i,y}} \mathbf{w}_u \\ \frac{\partial \mathbf{w}_u}{\partial \mathbf{x}_{i,z}} \mathbf{w}_u \end{pmatrix} \frac{A}{\left\| \mathbf{w}_v \right\|} \begin{pmatrix} \frac{\partial \mathbf{w}_v}{\partial \mathbf{x}_{i,x}} \mathbf{w}_v \\ \frac{\partial \mathbf{w}_v}{\partial \mathbf{x}_{i,y}} \mathbf{w}_v \\ \frac{\partial \mathbf{w}_v}{\partial \mathbf{x}_{i,z}} \mathbf{w}_v \end{pmatrix} \right) \begin{pmatrix} \mathbf{C}_{st,u} \\ \mathbf{C}_{st,v} \end{pmatrix}$$

Derivatives of **w**

- derivatives of $\mathbf{w}_u$ and $\mathbf{w}_v$ with respect to $\mathbf{x}_i$, $\mathbf{x}_j$, $\mathbf{x}_k$

$$\frac{\partial \mathbf{w}_u}{\partial \mathbf{x}_i} = \frac{\Delta v_1 - \Delta v_2}{\Delta u_1 \Delta v_2 - \Delta u_2 \Delta v_1} \mathbf{I}^{3x3} \qquad \frac{\partial \mathbf{w}_v}{\partial \mathbf{x}_i} = \frac{\Delta u_2 - \Delta u_1}{\Delta u_1 \Delta v_2 - \Delta u_2 \Delta v_1} \mathbf{I}^{3x3}$$

$$\frac{\partial \mathbf{w}_u}{\partial \mathbf{x}_j} = \frac{\Delta v_2}{\Delta u_1 \Delta v_2 - \Delta u_2 \Delta v_1} \mathbf{I}^{3x3} \qquad \frac{\partial \mathbf{w}_v}{\partial \mathbf{x}_j} = \frac{-\Delta u_2}{\Delta u_1 \Delta v_2 - \Delta u_2 \Delta v_1} \mathbf{I}^{3x3}$$

$$\frac{\partial \mathbf{w}_u}{\partial \mathbf{x}_k} = \frac{-\Delta v_1}{\Delta u_1 \Delta v_2 - \Delta u_2 \Delta v_1} \mathbf{I}^{3x3} \qquad \frac{\partial \mathbf{w}_v}{\partial \mathbf{x}_k} = \frac{\Delta u_1}{\Delta u_1 \Delta v_2 - \Delta u_2 \Delta v_1} \mathbf{I}^{3x3}$$

Stretch Summary

- stretch is represented with partial derivatives of a pseudo-mapping function which converts from initial planar coordinates to world coordinates
- stretch is considered in different directions
- stretch condition is stated per triangle
- stretch energy and stretch forces are derived from the cond.
- stretch forces for mass points of a triangle are computed using their positions in world space and their original positions in planar coordinates

Shearing

- shear of a triangle is measured by considering $\mathbf{w}_u^{\mathrm{T}}\mathbf{w}_v$
- in rest state this product is zero
  both vectors are orthogonal
- if shear occurs, the scalar condition function C_{sh} equals the cosine of the angle between $\mathbf{w}_u$ and $\mathbf{w}_v$ weighted by the initial triangle area A

$$C_{sh}(\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k) = A\mathbf{w}_u^{\mathrm{T}}\mathbf{w}_v$$

- $\mathbf{w}_u$ and $\mathbf{w}_v$ are not normalized, assuming their magnitudes do not change significantly due to stretch forces

Shear Force

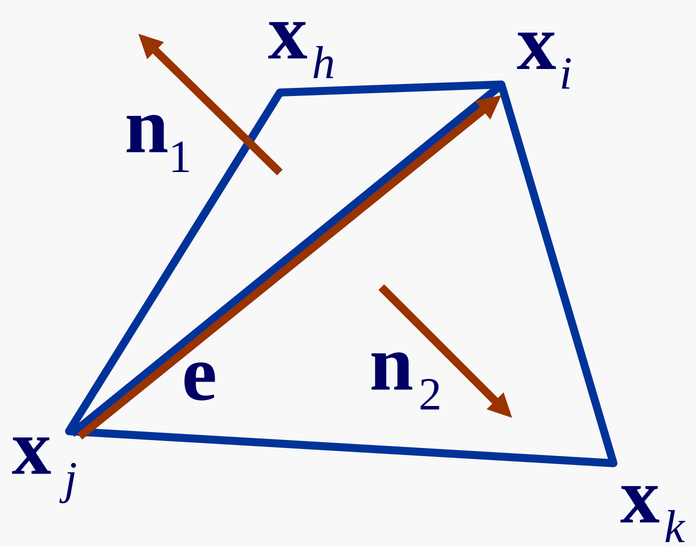
- force is derived from shear energy, user-defined k_{sh}
- forces at three triangle vertices are computed
  based on their positions in both object representations

$$\mathbf{F}_{sh}^{i} = -k_{sh} \frac{\partial C_{sh}}{\partial \mathbf{x}_i} C_{sh} = -k_{sh} \begin{pmatrix} \frac{\partial C_{sh}}{\partial \mathbf{x}_{i,x}} \\ \frac{\partial C_{sh}}{\partial \mathbf{x}_{i,y}} \\ \frac{\partial C_{sh}}{\partial \mathbf{x}_{i,z}} \end{pmatrix} C_{sh} = -k_{sh} A \begin{pmatrix} \frac{\partial \mathbf{w}_u}{\partial \mathbf{x}_{i,x}} \mathbf{w}_v + \frac{\partial \mathbf{w}_v}{\partial \mathbf{x}_{i,x}} \mathbf{w}_u \\ \frac{\partial \mathbf{w}_u}{\partial \mathbf{x}_{i,y}} \mathbf{w}_v + \frac{\partial \mathbf{w}_v}{\partial \mathbf{x}_{i,y}} \mathbf{w}_u \\ \frac{\partial \mathbf{w}_u}{\partial \mathbf{x}_{i,z}} \mathbf{w}_v + \frac{\partial \mathbf{w}_v}{\partial \mathbf{x}_{i,z}} \mathbf{w}_u \end{pmatrix} C_{sh}$$

Bend

- bend is measured for pairs of adjacent triangles with one common edge

$$
\begin{aligned}
\mathbf{n}_1 &= (\mathbf{x}_j - \mathbf{x}_h) \times (\mathbf{x}_i - \mathbf{x}_h) \\
\mathbf{n}_2 &= (\mathbf{x}_i - \mathbf{x}_k) \times (\mathbf{x}_j - \mathbf{x}_k) \\
\mathbf{e} &= \mathbf{x}_i - \mathbf{x}_j
\end{aligned}
$$



Bend Condition

- bend condition is the angle between a triangle pair

$$C_{be}(\mathbf{x}_h, \mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k) = \theta$$

- angle can be computed using

$$\sin\theta = (\mathbf{n}_1 \times \mathbf{n}_2) \cdot \mathbf{e}$$
$$\cos\theta = \mathbf{n}_1 \cdot \mathbf{n}_2$$

- again, normalization is neglected, assuming the magnitudes do not change significantly

Bend Forces

- force is derived from bend energy with user-defined k_{be}

$$\mathbf{F}_{be}^{i} = -k_{be} \frac{\partial C_{be}}{\partial \mathbf{x}_i} C_{be} = -k_{be} \begin{pmatrix} \frac{\partial C_{be}}{\partial \mathbf{x}_{i,x}} \\ \frac{\partial C_{be}}{\partial \mathbf{x}_{i,y}} \\ \frac{\partial C_{be}}{\partial \mathbf{x}_{i,z}} \end{pmatrix} C_{be}$$

- no weighting with triangle area
- implicitly considered by using unnormalized vectors. However, normalized vectors are used.

Bend Forces - Derivatives

- to compute $\frac{\partial C_{be}}{\partial \mathbf{x}_i}$ both relations for the angle are used due to singularities

$$\frac{\partial \cos\theta}{\partial \mathbf{x}} = -\sin\theta \frac{\partial \theta}{\partial \mathbf{x}} = \frac{\partial \mathbf{n}_1 \cdot \mathbf{n}_2}{\partial \mathbf{x}}$$

$$\rightarrow \quad \frac{\partial \theta}{\partial \mathbf{x}} = \frac{-1}{\sin\theta} \frac{\partial \mathbf{n}_1 \cdot \mathbf{n}_2}{\partial \mathbf{x}} = \frac{-1}{(\mathbf{n}_1 \times \mathbf{n}_2) \cdot \mathbf{e}} \frac{\partial \mathbf{n}_1 \cdot \mathbf{n}_2}{\partial \mathbf{x}}$$

$$\frac{\partial \sin\theta}{\partial \mathbf{x}} = \cos\theta \frac{\partial \theta}{\partial \mathbf{x}} = \frac{\partial (\mathbf{n}_1 \times \mathbf{n}_2) \cdot \mathbf{e}}{\partial \mathbf{x}}$$

$$\rightarrow \quad \frac{\partial \theta}{\partial \mathbf{x}} = \frac{1}{\cos\theta} \frac{\partial (\mathbf{n}_1 \times \mathbf{n}_2) \cdot \mathbf{e}}{\partial \mathbf{x}} = \frac{1}{\mathbf{n}_1 \cdot \mathbf{n}_2} \frac{\partial (\mathbf{n}_1 \times \mathbf{n}_2) \cdot \mathbf{e}}{\partial \mathbf{x}}$$

Bend Forces - Derivatives

$$\mathbf{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} \rightarrow \tilde{\mathbf{n}} = \begin{pmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{pmatrix}$$

$$\mathbf{n}_1 \times \mathbf{n}_2 = \tilde{\mathbf{n}}_1 \cdot \mathbf{n}_2 = \begin{pmatrix} 0 & -n_{1,z} & n_{1,y} \\ n_z & 0 & -n_{1,x} \\ -n_{1,y} & n_{1,x} & 0 \end{pmatrix} \begin{pmatrix} n_{2,x} \\ n_{2,y} \\ n_{2,z} \end{pmatrix} = \begin{pmatrix} n_{1,y} n_{2,z} - n_{2,y} n_{1,z} \\ n_{1,z} n_{2,x} - n_{2,z} n_{1,x} \\ n_{1,x} n_{2,y} - n_{2,x} n_{1,y} \end{pmatrix}$$

$$\frac{\partial(\mathbf{n}_1 \times \mathbf{n}_2)}{\partial \mathbf{x}} = \frac{\partial(\tilde{\mathbf{n}}_1 \cdot \mathbf{n}_2)}{\partial \mathbf{x}} = \begin{pmatrix} \widetilde{\left(\frac{\partial \mathbf{n}_1}{\partial x_x}\right)} \mathbf{n}_2 - \widetilde{\left(\frac{\partial \mathbf{n}_2}{\partial x_x}\right)} \mathbf{n}_1 \\ \widetilde{\left(\frac{\partial \mathbf{n}_1}{\partial x_y}\right)} \mathbf{n}_2 - \widetilde{\left(\frac{\partial \mathbf{n}_2}{\partial x_y}\right)} \mathbf{n}_1 \\ \widetilde{\left(\frac{\partial \mathbf{n}_1}{\partial x_z}\right)} \mathbf{n}_2 - \widetilde{\left(\frac{\partial \mathbf{n}_2}{\partial x_z}\right)} \mathbf{n}_1 \end{pmatrix}$$

Bend Forces - Derivatives

- derivatives of **e**

$$\frac{\partial \mathbf{e}}{\partial \mathbf{x}_i} = \frac{\partial \left(\mathbf{x}_i - \mathbf{x}_j \right)}{\partial \mathbf{x}_i} = \mathbf{I}^{3x3}$$

$$\frac{\partial \mathbf{e}}{\partial \mathbf{x}_j} = \frac{\partial \left(\mathbf{x}_i - \mathbf{x}_j \right)}{\partial \mathbf{x}_j} = -\mathbf{I}^{3x3}$$

Bend Forces - Derivatives

- derivatives of **n**

$$\frac{\partial \mathbf{n}_1}{\partial \mathbf{x}_h} = \frac{\partial\left[\left(\mathbf{x}_j - \mathbf{x}_h\right)\times\left(\mathbf{x}_i - \mathbf{x}_h\right)\right]}{\partial \mathbf{x}_h} = \begin{pmatrix} \widetilde{(-1\;\;0\;\;0)}(\mathbf{x}_i - \mathbf{x}_h) - \widetilde{(-1\;\;0\;\;0)}(\mathbf{x}_j - \mathbf{x}_h) \\ \widetilde{(0\;\;-1\;\;0)}(\mathbf{x}_i - \mathbf{x}_h) - \widetilde{(0\;\;-1\;\;0)}(\mathbf{x}_j - \mathbf{x}_h) \\ \widetilde{(0\;\;0\;\;-1)}(\mathbf{x}_i - \mathbf{x}_h) - \widetilde{(0\;\;0\;\;-1)}(\mathbf{x}_j - \mathbf{x}_h) \end{pmatrix}$$

$$\frac{\partial \mathbf{n}_1}{\partial \mathbf{x}_h} = \begin{pmatrix} \widetilde{(-1\;\;0\;\;0)}(\mathbf{x}_i - \mathbf{x}_j) \\ \widetilde{(0\;\;-1\;\;0)}(\mathbf{x}_i - \mathbf{x}_j) \\ \widetilde{(0\;\;0\;\;-1)}(\mathbf{x}_i - \mathbf{x}_j) \end{pmatrix} = \widetilde{(\mathbf{x}_j - \mathbf{x}_i)}$$

Bend Forces - Derivatives

- derivatives of **n**

$$\frac{\partial \mathbf{n}_1}{\partial \mathbf{x}_h} = \widetilde{(\mathbf{x}_j - \mathbf{x}_i)} \qquad \frac{\partial \mathbf{n}_1}{\partial \mathbf{x}_i} = \widetilde{(\mathbf{x}_h - \mathbf{x}_j)} \qquad \frac{\partial \mathbf{n}_1}{\partial \mathbf{x}_j} = \widetilde{(\mathbf{x}_i - \mathbf{x}_h)}$$

$$\frac{\partial \mathbf{n}_2}{\partial \mathbf{x}_i} = \widetilde{(\mathbf{x}_j - \mathbf{x}_k)} \qquad \frac{\partial \mathbf{n}_2}{\partial \mathbf{x}_j} = \widetilde{(\mathbf{x}_k - \mathbf{x}_i)} \qquad \frac{\partial \mathbf{n}_2}{\partial \mathbf{x}_k} = \widetilde{(\mathbf{x}_i - \mathbf{x}_j)}$$

Bend Forces – Last Derivatives

- derivatives of the dot and the vector product

$$\mathbf{x} \in \{\mathbf{x}_h, \mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k\}$$

$$\frac{\partial \mathbf{n}_1 \cdot \mathbf{n}_2}{\partial \mathbf{x}} = \frac{\partial \mathbf{n}_1}{\partial \mathbf{x}} \mathbf{n}_2 + \frac{\partial \mathbf{n}_2}{\partial \mathbf{x}} \mathbf{n}_1$$

$$\frac{\partial (\mathbf{n}_1 \times \mathbf{n}_2) \cdot \mathbf{e}}{\partial \mathbf{x}} = \frac{\partial (\mathbf{n}_1 \times \mathbf{n}_2)}{\partial \mathbf{x}} \mathbf{e} + \frac{\partial \mathbf{e}}{\partial \mathbf{x}} (\mathbf{n}_1 \times \mathbf{n}_2)$$

Simulation Loop

- initialization
  - generation of the initial planar triangle mesh
  - definition of (u , v) and (x , y , z) coordinates for all vertices
  - definition of the initial velocity for all vertices
  - definition of mass for all vertices
  - definition of k_{st} , k_{sh} for all triangles
  - definition of k_{be} for all triangle pairs
- forces
  - loop triangles: compute $\mathbf{F}_{st}$, $\mathbf{F}_{sh}$ for the three vertices
  - loop pairs of triangles: compute F_{be} for the four vertices
  - loop vertices: compute external force
- apply your favorite numerical integration scheme

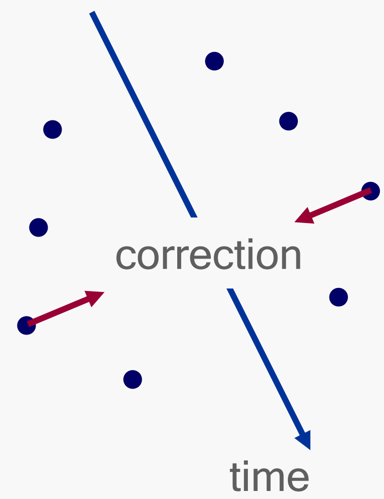
Experimental Results

- Pritchard
  - $k_{st} = 0.5 \cdot 10^4$, $k_{sh} = 0.5 \cdot 10^3$, $k_{be} = 0.1 \cdot 10^{-4}$
  - time step = 0.02s
  - implicit integration
  - 66 x 66 mass points
  - computing time 45 s per simulation step (Pentium 4, 3GHz)
- Baraff, Witkin
  - 4500 mass points, 10 s per simulation step
- both implementations use implicit integration and conjugate gradients with an unkown number of iterations

Strain Limiting

[Provot, Bridson]

- distance deviation of two adjacent points limited to 10% of the initial distance
- larger deviations are prevented by symmetrically replacing both points
- realized by looping through all point pairs
- convergent if performed iteratively
- replacing can be interpreted as the result of an internal force $\rightarrow$ global dynamic behavior (linear, angular momentum of the model) is not influenced



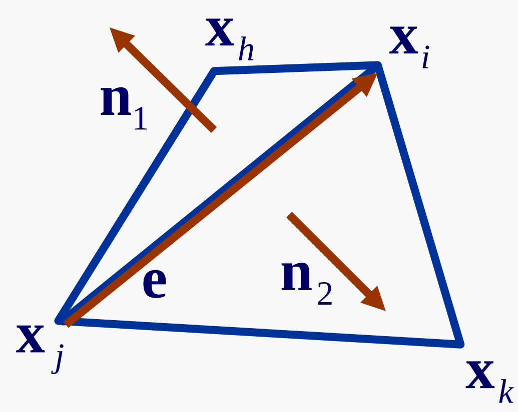
Strain Limiting 2

[Bridson, Fedkiw]

- adjustment of velocities instead of positions
- in the simulation loop
  - predict new velocities
  - if positions based on the predicted velocities exceed the strain limit, correct velocities accordingly
  - compute new positions
- computed positions based on corrected velocities fulfill the strain limit
- symmetric changes of velocities maintain angular and linear momentum of the system
- consistent positions and velocities

Alternative Bend Forces

[Bridson, Fedkiw]



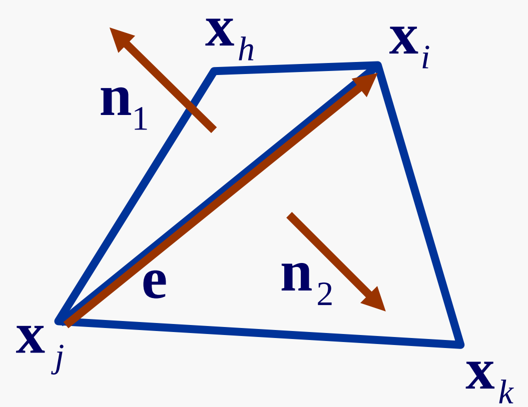
- derivation of forces based on the bending angle between $\mathbf{n}_1$ and $\mathbf{n}_2$
- forces should not cause rigid-body motion of the four points and should not cause in-plane deformations (compare def. of internal forces)
- $\mathbf{u}_h$, $\mathbf{u}_i$, $\mathbf{u}_j$, $\mathbf{u}_k$ are force directions at $\mathbf{x}_h$, $\mathbf{x}_i$, $\mathbf{x}_j$, $\mathbf{x}_k$
- $\mathbf{u}_h$ has to be parallel to $\mathbf{n}_1$, $\mathbf{u}_k$ has to be parallel to $\mathbf{n}_2$
- $\mathbf{u}_i$, $\mathbf{u}_j$ have to be in the span of $\mathbf{n}_1$ and $\mathbf{n}_2$
- $\mathbf{u}_h+\mathbf{u}_i+\mathbf{u}_j+\mathbf{u}_k=0$, no change of linear velocity
- $(\mathbf{u}_h, \mathbf{u}_i, \mathbf{u}_j, \mathbf{u}_k)$ has to be orthogonal to rigid body rotation

Force Directions

$$\mathbf{u}_h = |\mathbf{e}| \frac{\mathbf{n}_1}{|\mathbf{n}_1|^2} \qquad \mathbf{u}_k = |\mathbf{e}| \frac{\mathbf{n}_2}{|\mathbf{n}_2|^2}$$

$$\mathbf{u}_j = \frac{(\mathbf{x}_h - \mathbf{x}_i)\cdot \mathbf{e}}{|\mathbf{e}|} \frac{\mathbf{n}_1}{|\mathbf{n}_1|^2} + \frac{(\mathbf{x}_k - \mathbf{x}_i)\cdot \mathbf{e}}{|\mathbf{e}|} \frac{\mathbf{n}_2}{|\mathbf{n}_2|^2}$$

$$\mathbf{u}_i = -\frac{(\mathbf{x}_h - \mathbf{x}_j)\cdot \mathbf{e}}{|\mathbf{e}|} \frac{\mathbf{n}_1}{|\mathbf{n}_1|^2} - \frac{(\mathbf{x}_k - \mathbf{x}_j)\cdot \mathbf{e}}{|\mathbf{e}|} \frac{\mathbf{n}_2}{|\mathbf{n}_2|^2}$$



Force Magnitude

$$\mathbf{F}_{be}^{\{h,i,j,k\}} = k_{be} \frac{|\mathbf{e}|^2}{|\mathbf{n}_1| + |\mathbf{n}_2|} \sin(\theta/2) \mathbf{u}_{\{h,i,j,k\}}$$

- k_{be} is user-defined
- additional heuristic weighting with triangle areas ensures independence from meshing
- sin is just simply to compute

$$\sin(\theta/2) = \pm\sqrt{(1 - \mathbf{n}_1 \cdot \mathbf{n}_2)/2}$$

Comparison

[Baraff, Witkin] - [Bridson, Fedkiw]

- Baraff, Witkin define conditions
- principle of energy-driven forces ensures that resulting forces are internal forces
- elegant, but not straight-forward to implement

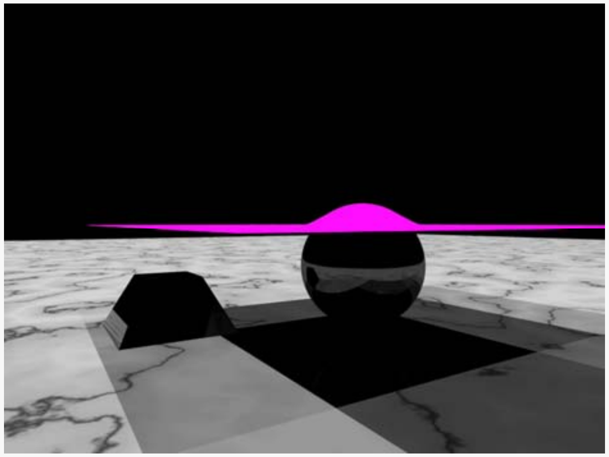
- Bridson, Fedkiw define forces
- forces are designed to represent resistance to bending
- forces are designed to be internal forces
- less elegant, but practical

Related Approaches

- link list at the University College London http://www.cs.ucl.ac.uk/research/vr/Projects/3DCentre/cloth_simulation_links.htm

Volino, Thalmann

Eberhardt, Weber, Strasser



Bridson, Fedkiw, Anderson

Choi, Ko

Commercial Cloth Simulation

- www.syflex.biz
- “Our cloth simulator implements a completely original technique.” syflex.

syflex

Beyond Skirts - Open Problems

- realistic materials – are resistances to shear, stretch, bending an appropriate parameterization?
- stable collision response in case of collisions and self-collisions (tight-fitting cloth)

sharp edges
layered material

wrinkles

men