On Multiple Virtues of Blue Noise Sampling

Victor Ostromoukhov
University of Lyon/CNRS

Joint Work with David Coeurjolly, Adrien Pilleboue, Gurprit Singh, Helene Perrier, Abdalla Ahmed, Eric Heitz, Laurent Belcour, Matt Pharr, Michael Kazhdan, Jianwei Guo, Dongming Yan, Hui Huang, Oliver Deussen, Feng Xie, Pat Hanrahan
ABDALLA AHMED
MOTIVATION

- BN sampling is good for diminishing overall noise in MC integration
- BN sampling is good for improving visual appearance of synthetic images
- Advanced BN sampling can be efficiently implemented
OVERVIEW

- Theoretical foundation for BN sampling
  Based on Variance Analysis for Monte Carlo Integration, SIGGRAPH 2015
- Some efficient implementations
  Based on
  Low-Discrepancy Blue Noise Sampling, SIGGRAPH-ASIA 2016
  Sequences with Low-Discrepancy Blue-Noise 2-D Projections, EG2018
  A Low-Discrepancy Sampler that Distributes Monte Carlo Errors as a Blue Noise in Screen Space, SIGGRAPH 2019 Talk

Open Issues
BLUE NOISE

Digital Halftoning
Robert Ulichney

[Ulichney 1987]
BN IN NATURE: COMPETITION FOR THE VITAL SPACE
BN (ORGANIC) VS. ARTIFICIAL (ORDERED) DISTRIBUTIONS

BN Points
[de Goes et al. 2012]

Sobol Points
BLUE NOISE POWER AND RADIAL SPECTRA IN 2D

[de Goes et al. 2012]
BN: TARGET BEHAVIOR OF MSE IN INTEGRATION
THEORETICAL FOUNDATION FOR BN SAMPLING

Variance Analysis for Monte Carlo Integration

Abstract. We propose a new spectral analysis of the variance in Monte Carlo integration, expressed in terms of the power spectra of the sampling pattern and the integrand involved. We build our framework in the Euclidean space using Fourier tools and on the sphere using spherical harmonics. We further provide a theoretical background that explains how our spherical framework can be extended to the hemispherical domain. We use our framework to estimate the variance convergence rate of different state-of-the-art sampling patterns in both the Euclidean and spherical domains, as the number of samples increases. Furthermore, we formulate design principles for constructing sampling methods that can be tailored according to available resources. We validate our theoretical framework by performing numerical integration over several integrands sampled using different sampling patterns.

VARIANCE FORMULATION BASED ON FOURIER ANALYSIS

\[ \text{Var}(I_N) = \frac{\mu(T^d)\mu(S^{d-1})}{N} \int_0^\infty \rho^{d-1} \tilde{P}_S(\rho)\tilde{P}_F(\rho) d\rho \]

(Jittered Sampling Pattern)
VARIANCE FORMULATION BASED ON FOURIER ANALYSIS
TAXONOMY OF CONVERGENCE CLASSES

\[ b = 0 \]

Best-case: \( O \left( \frac{1}{N} \right) \)

Worst-case: \( O \left( \frac{1}{N} \right) \)

\[ 0 < b \leq 1 \]

Best-case: \( O \left( \frac{1}{N^{d/N^b}} \right) \)

Worst-case: \( O \left( \frac{1}{N^{d/N^b}} \right) \)

\[ b \geq 1 \]

Best-case: \( O \left( \frac{1}{N^{d/N^b}} \right) \)

Worst-case: \( O \left( \frac{1}{N^{d/N^b}} \right) \)

\[ b \to \infty \]

Best-case: \( 0 \)

Worst-case: \( O \left( \frac{1}{N^{d/N}} \right) \)

b: degree of the polynomial

d: dimensions

N: number of samples
LOW FREQUENCY REGION

Poisson Disk: $O\left(\frac{1}{N}\right)$

Jittered: $O\left(\frac{1}{N^{1/2}}\right)$
VERIFICATION OF THE THEORETICAL PREDICTION

(a) Jittered sampling

(b) Variance in MC integration of a Gaussian

(c) Variance in MC integration of a disk

(d) Poisson disk
OVERVIEW

- Theoretical foundation for BN sampling
  Based on Variance Analysis for Monte Carlo Integration, SIGGRAPH 2015

- Some efficient implementation
  Based on
  - Low-Discrepancy Blue Noise Sampling, SIGGRAPH-ASIA 2016
  - Sequences with Low-Discrepancy Blue-Noise 2-D Projections, EG2018
  - A Low-Discrepancy Sampler that Distributes Monte Carlo Errors as a Blue Noise in Screen Space, SIGGRAPH 2019 Talk

- Open Issues
Low-Discrepancy Blue Noise Sampling

Abdalla G. M. Ahmed\textsuperscript{1} 
Jianwei Guo\textsuperscript{3}

Hélène Perrier\textsuperscript{2} 
Dongming Yan\textsuperscript{3} 
David Coeurjolly\textsuperscript{2} 
Hui Huang\textsuperscript{4,5} 
Victor Ostromoukhov\textsuperscript{2} 
Oliver Deussen\textsuperscript{1,5}

\textsuperscript{1}University of Konstanz, Germany 
\textsuperscript{2}Université de Lyon, CNRS/LIRIS, France 
\textsuperscript{3}NLPR, Institute of Automation, CAS, China 
\textsuperscript{4}Shenzhen University, China 
\textsuperscript{5}SIAT, China

---

Graph showing the comparison of various noise generation methods with the proposed method.
2D INDEXED LD SETS
DISCREPANCY-PRESERVING REARRANGEMENT
AXIS-WISE 2D REARRANGEMENT DEMO
REFERENCE-MATCHING ALGORITHM
Sequences with Low-Discrepancy Blue-Noise 2-D Projections

Hélène Perrier¹, David Coeurjolly¹, Feng Xie², Matt Pharr³, Pat Hanrahan², Victor Ostromoukhov¹

¹Université de Lyon, CNRS, LIRIS, France ²Stanford, USA ³Google, USA

In Computer Graphics Forum (Proceedings of Eurographics), 2018

Left: our staged per-tile optimized scrambling, applied to a Sobol sequence of sampling points, produces a power spectrum close to Blue Noise. Right: Rendering of a challenging scene featuring depth of field and high specularity (jewels). The sampling was done with the Sobol sequence (top) and our sampler (bottom); both use 256 samples per pixel and 3 light bounces. Note the improvement in aliasing when using our method in comparison to the original Sobol sequence.
OWEN’S SCRAMBLING
OWEN’S SCRAMBLING

\{0\}, \{1, 1\}, \{1, 0, 0, 1\}, \{1, 0, 0, 1, 0, 1, 1\}

\{1\}, \{1, 1\}, \{1, 0, 1, 1\}, \{0, 0, 0, 0, 1, 1, 0\}
OWEN’S SCRAMBLING
<table>
<thead>
<tr>
<th>Point Distribution</th>
<th>Fourier Spectrum</th>
<th>Radial Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sobol</td>
<td><img src="image1.png" alt="Sobol Point Distribution" /></td>
<td><img src="image2.png" alt="Sobol Fourier Spectrum" /></td>
</tr>
<tr>
<td>Owen’s scrambling</td>
<td><img src="image4.png" alt="Owen’s Point Distribution" /></td>
<td><img src="image5.png" alt="Owen’s Fourier Spectrum" /></td>
</tr>
</tbody>
</table>
CONSTRUCTION IN [PERRIER ET AL. 2018]: THE KEY IDEA

Step 1: Identify all possible 16-point tiles in the original Sobol sets
CONSTRUCTION IN [PERRIER ET AL. 2018]: THE KEY IDEA

Step 2: For each ID, find Owen’s permutations which maximize min dist
CONSTRUCTION IN [PERRIER ET AL. 2018]: THE KEY IDEA

Step 2: For each ID, find Owen’s permutations which maximize min dist

Dyadic Partitioning is preserved:
CONSTRUCTION IN [PERRIER ET AL. 2018]: THE KEY IDEA

Step 3: Store permuted patterns in a lookup table

Step 4: In runtime, take the pattern from the lookup table, and LSB bits from Sobol’s codes:
GENERATED POINTS (4K): SOBOL
GENERATED POINTS (4K): OWEN’S SCRAMBLING
GENERATED POINTS (4K): [PERRIER ET AL. 2018]
POWER SPECTRUM + RADIAL: OWEN VS. [PERRIER ET AL. 2018]
<table>
<thead>
<tr>
<th>Point Distribution</th>
<th>Fourier Spectrum</th>
<th>Radial Averaged Power Spectra</th>
<th>Zone Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicked (gray)</td>
<td><img src="image" alt="Fourier Spectrum" /></td>
<td><img src="image" alt="Radial Averaged Power Spectra" /></td>
<td><img src="image" alt="Zone Plate" /></td>
</tr>
<tr>
<td>Reused</td>
<td><img src="image" alt="Fourier Spectrum" /></td>
<td><img src="image" alt="Radial Averaged Power Spectra" /></td>
<td><img src="image" alt="Zone Plate" /></td>
</tr>
<tr>
<td>Overlapping</td>
<td><img src="image" alt="Fourier Spectrum" /></td>
<td><img src="image" alt="Radial Averaged Power Spectra" /></td>
<td><img src="image" alt="Zone Plate" /></td>
</tr>
<tr>
<td>Overlap (2x2)</td>
<td><img src="image" alt="Fourier Spectrum" /></td>
<td><img src="image" alt="Radial Averaged Power Spectra" /></td>
<td><img src="image" alt="Zone Plate" /></td>
</tr>
<tr>
<td>Overlap (2x2)</td>
<td><img src="image" alt="Fourier Spectrum" /></td>
<td><img src="image" alt="Radial Averaged Power Spectra" /></td>
<td><img src="image" alt="Zone Plate" /></td>
</tr>
<tr>
<td>Overlap (2x2)</td>
<td><img src="image" alt="Fourier Spectrum" /></td>
<td><img src="image" alt="Radial Averaged Power Spectra" /></td>
<td><img src="image" alt="Zone Plate" /></td>
</tr>
<tr>
<td>Overlap (2x2)</td>
<td><img src="image" alt="Fourier Spectrum" /></td>
<td><img src="image" alt="Radial Averaged Power Spectra" /></td>
<td><img src="image" alt="Zone Plate" /></td>
</tr>
<tr>
<td>Overlap (2x2)</td>
<td><img src="image" alt="Fourier Spectrum" /></td>
<td><img src="image" alt="Radial Averaged Power Spectra" /></td>
<td><img src="image" alt="Zone Plate" /></td>
</tr>
<tr>
<td>Overlap (2x2)</td>
<td><img src="image" alt="Fourier Spectrum" /></td>
<td><img src="image" alt="Radial Averaged Power Spectra" /></td>
<td><img src="image" alt="Zone Plate" /></td>
</tr>
</tbody>
</table>
[PERRIER ET AL. 2018]: CONCLUSIONS

What we Got:
- 2-D Low-Discrepancy Sequences (with Support for Progressive Sampling)
- Improved 2-D Fourier Spectra
- Extendable to 4-D and 6-D
- Supports Adaptive Sampling
- Fast, Low Memory Footprint
- Purely Deterministic, but Can Simulates Quasi-Randomness

Limitations:
- Hard to Get Higher Dimensions
- Power Spectra are “Blueish” rather then “Blue”
A Low-Discrepancy Sampler that Distributes Monte Carlo Errors as a Blue Noise in Screen Space

Eric Heitz\(^1\) Laurent Belcour\(^1\) Victor Ostromoukhov\(^2\) David Coeurjolly\(^2\) Jean-Claude Iehl\(^2\)

\(^1\)Unity Technologies \(^2\)Université de Lyon, CNRS, LIRIS, France

In ACM SIGGRAPH Talk, 2019
OVERVIEW

- Theoretical foundation for BN sampling
  Based on Variance Analysis for Monte Carlo Integration, SIGGRAPH 2015

- Some efficient implementation
  Based on
  Low-Discrepancy Blue Noise Sampling, SIGGRAPH-ASIA 2016
  Sequences with Low-Discrepancy Blue-Noise 2-D Projections, EG2018
  A Low-Discrepancy Sampler that Distributes Monte Carlo Errors as a
  Blue Noise in Screen Space, SIGGRAPH 2019 Talk

- Open Issues
WHAT IS WRONG WITH DISCREPANCY AS MEASURE OF UNIFORMITY?

Disk function: sampling error

Triangle function: sampling error

[Christensen et al. 2018]
WHAT IS WRONG WITH DISCREPANCY AS MEASURE OF UNIFORMITY?

Triangle function: sampling error

Sobol Points
WHAT IS WRONG WITH DISCREPANCY AS MEASURE OF UNIFORMITY?
MY FAVORITE SAMPLER?
MY FAVORITE SAMPLER?

The future one!
MY FAVORITE SAMPLER?

The future one!

- Multi-dimensional
- Guarantees convergence to the true integral
- Prevents aliasing
- Minimizes noise
- Guarantees good frequency content of the noise
- Guarantees good computational efficiency
QUESTIONS?