MLS Pressure Boundaries for Divergence-Free and Viscous SPH Fluids - Appendix

Stefan Band

Mass of a boundary particle Iterative solvers such as PCISPH [9], IISPH [6] or DFSPH [5] compute a pressure field p and apply pressure accelerations of the form

$$\boldsymbol{a}_f^{\mathrm{p}} = -\sum_j m_j \left(\frac{p_f}{\rho_f^2} + \frac{p_j}{\rho_j^2} \right) \boldsymbol{\nabla} W_{fj} - \sum_b m_b \left(\frac{p_f}{\rho_f^2} + \frac{p_b}{\rho_b^2} \right) \boldsymbol{\nabla} W_{ff_b} \tag{1}$$

to fluid particles f [4]. Here, j and b denote fluid and boundary neighbors of fluid particle f, respectively. Equation (1) requires a notion of mass m_b and density ρ_b at boundary particles b.

The mass m_b at a boundary particle b can be geometrically motivated from the boundary particle volume V_b [2, 8, 3], i.e. the mass m_b can be derived from the relation $m_b = \rho_b V_b$. Since the density ρ_b is typically set to the rest density of the adjacent fluid particle [2], i.e.

$$\rho_b = \rho_f \ , \tag{2}$$

the mass m_b of a boundary particle b can be computed as

$$m_b = \rho_f^0 V_b^0 \ , \tag{3}$$

where ρ_f^0 denotes the rest density of fluid particle f and the rest volume V_b^0 is [3]

$$V_b^0 = \frac{\gamma}{\sum_{b_b} W_{bb_b}} \ . \tag{4}$$

The computation of Eq. (4) only processes boundary neighbors b_b of a boundary particle b as the rest volume does not depend on possibly adjacent fluid particles [3]. As only one layer of boundary particles is used to represent the surface of the boundary [2], the coefficient γ accounts for an incomplete neighborhood. The coefficient depends on the choice of the SPH kernel function. For the cubic spline kernel [7] with a smoothing length of two times the particle size h, $\gamma \approx 0.7$ [3]. This motivated by the fact that $0.7/\sum_{b_b} W_{bb_b} = h^3$ for boundary particles that are evenly sampled in a plane. See Fig. 1 for an illustration of the derivation.

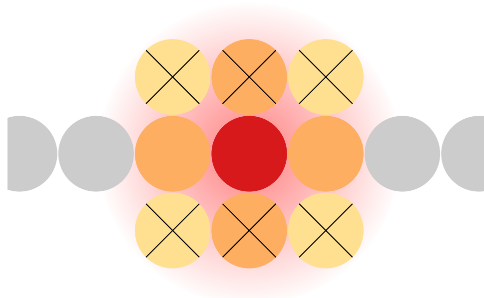

Figure 1: Cross section of a boundary plane with color-coded SPH kernel weights. In 3D, red represents 31.8 %, orange 8 % and yellow 1.6 % of the total kernel weight. Due to missing particle neighbors (crossed), the sum of kernel weights of the central particle is not 100 % but $1 \cdot 31.8\,\% + 4 \cdot 8\,\% + 4 \cdot 1.6\,\% = 70.2\,\%$.

Implementation Employing Eqs. (2) to (4), the pressure acceleration of a fluid particle f, Eq. (1), can be computed as

$$\boldsymbol{a}_f^{\mathrm{p}} = -\sum_j m_j \left(\frac{p_f}{\rho_f^2} + \frac{p_j}{\rho_j^2} \right) \boldsymbol{\nabla} W_{fj} - \sum_b \rho_f^0 V_b^0 \left(\frac{p_f}{\rho_f^2} + \frac{p_b}{\rho_f^2} \right) \boldsymbol{\nabla} W_{fb} \,. \qquad (5)$$

References

[1] S. Adami, X. Y. Hu, and N. A. Adams. A generalized wall boundary condition for smoothed particle hydrodynamics. *Journal of Computational Physics*, 231(21):7057 – 7075, 2012.

[2] N. Akinci, M. Ihmsen, G. Akinci, B. Solenthaler, and M. Teschner. Versatile Rigid-fluid Coupling for Incompressible SPH. *ACM Transactions on Graphics*, 31(4):62:1–62:8, 2012.

[3] S. Band, C. Gissler, M. Ihmsen, J. Cornelis, A. Peer, and M. Teschner. Pressure Boundaries for Implicit Incompressible SPH. *ACM Transactions on Graphics*, 37(2):14:1–14:11, 2018. Presented at SIGGRAPH 2018.

[4] S. Band, C. Gissler, A. Peer, and M. Teschner. MLS Pressure Boundaries for Divergence-Free and Viscous SPH Fluids. *Computers & Graphics*, 76:37–46, 2018.

[5] J. Bender and D. Koschier. Divergence-Free SPH for Incompressible and Viscous Fluids. *IEEE Transactions on Visualization and Computer Graphics*, 23(3):1193–1206, 2017.

[6] M. Ihmsen, J. Cornelis, B. Solenthaler, C. Horvath, and M. Teschner. Implicit incompressible SPH. *IEEE Transactions on Visualization and Computer Graphics*, 20(3):426–435, 2014.

[7] J. J. Monaghan. Smoothed Particle Hydrodynamics. *Reports on Progress in Physics*, 68(8):1703, 2005.

[8] S. Rosswog. SPH Methods in the Modelling of Compact Objects. *Living Reviews in Computational Astrophysics*, 1(1), 2015.

[9] B. Solenthaler and R. Pajarola. Predictive-corrective Incompressible SPH. *ACM Transactions on Graphics*, 28(3):40:1–40:6, 2009.